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Evaluation of the Loran Tests at Anniston, Alabama, and Panama City, Florida

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BOULDER, COLO.
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EVALUATION OF THE LORAN TESTS AT ANNISTON, ALABAMA, AND PANAMA CITY, FLORIDA

Bernard Wieder and James S. Washburn

The results from an evaluation of the differential loran technique conducted in southeastern United States are discussed. The discussion centers on equipment performance and data error analysis. The results from the evaluation show relatively large systematic discrepancies from predicted loran coordinates. The analysis attempts to isolate the errors attributable to long-and short-range propagation effects from systematic errors in the manpack receivers and the loran chain. Recommendations are made for possible future tests to more clearly determine long- and short-range loran propagation effects for a given area.

Key Words: Loran-C, differential loran-C, ground wave propagation, manpack receivers, irregular terrain, inhomogeneous terrain

1. INTRODUCTION

The Naval Applied Science Laboratory (NASL) has been developing a "differential loran" technique for use in combat areas. Two (or more) loran navigation receivers are used, which are first compared with each other at a single site. One receiver remains at that site; the other is taken on the mission. By using the first (fixed) receiver as a calibration instrument and comparing the readings on the moving receiver with it, the errors caused by offsets and systematic temporal variations in the loran grid can be determined by the fixed receiver and the readings of the moving receiver appropriately adjusted to better determine position.

To implement the technique, NASL tested two manpack loran-C navigators (Electronics, 1968). Mr. David Pessin directed the differential loran program for NASL; Mr. Fred Pappalardi was in charge of the field tests. ESSA personnel participated in the tests and in the

evaluation of the data. This report presents some of the results of the ESSA evaluation.

2. BACKGROUND

Two test series were run, the first during May 1968 and the second during May 1969. Each series involved two test areas, one in the vicinity of Anniston, Alabama, and the other near Panama City, Florida. The Anniston area is characterized by heavily wooded rolling hills at a mean altitude of 600 to 700 ft, with occasional ridges rising to about 2000 ft. These ridges made it possible to investigate the effects on the differential loran measurements of loran grid anomalies due to irregular terrain. Table 1 lists the names of the sites used, all of which were located as near as possible to Coast and Geodetic Survey benchmarks for precise position determination. Two (Horn and Able) were on top of ridges near fire watch towers. Also given in table 1 are the latitudes and longitudes of the benchmarks and the predicted time difference readings that were expected for the loran triad, consisting of the master station at Cape Fear, North Carolina, and the slave stations at Jupiter Inlet, Florida, and Dana, Indiana. The predicted values are based on routine loran calculations with secondary phase corrections (Johler, Kellar, and Walters, 1956) based on sea water conductivity.

The Panama City area is lightly wooded, relatively flat, and near the seashore. The names and locations of the Florida sites appear in table 2.

To test the manpack receivers, TDA and TDB readings in microseconds were taken at all the sites. TDA is here defined as the time difference between the master station and slave A (Jupiter Inlet), TDB as the time difference between the master station and slave B (Dana). Miles are statute miles.

Table 1. Alabama Test Site.

Site	N. Latitude	W. Longitude	TDA (Master-Jupiter) (μsec)	TDB (Master-Dana) (μsec)
Delta	33°24' 03.59"	85°41' 34.69"	14236.66	68590.79
Taylor	33°33' 10.15"	85°39' 07.60"	14292.69	68558.35
TT-1	33°43' 08.60"	85°54' 17.12"	14317.93	68410.40
Airport	33°35' 27.12"	85°51' 23.23"	14281.63	68470.03
Horn	33°17' 52.01"	86°04' 28.79"	14160.56	68483.28
Able	33°33' 36.10"	85°41' 54.65"	14289.66	68538.64
Bynum	33°37' 03.71"	85°59' 08.01"	14275.80	68413.95
Piedmont	33°56' 0.79"	85°37' 01.14"	14421.94	68446.60
Mead	33°42' 36.74"	85°57' 51.34"	14308.21	68391.44

Table 2. Florida Test Site.

Site	N. Latitude	W. Longitude	TDA (Master-Jupiter) (μsec)	TDB (Master-Dana) (μsec)
Burnt	30°19' 53.42"	85°45' 01.54"	13075.76	69299.94
Southport	30°16' 47.39"	85°38' 45.95"	13055.73	69342.19
West	30°14' 50.24"	85°52' 50.44"	13043.17	69273.27
Goose	29°57' 50.84"	85°26' 43.13"	12929.66	69457.55
Park	30°08' 13.04"	85°44' 23.66"	13000.13	69336.52

3. AIRPORT TEST

The basic technique for taking readings was to observe repeated readouts of each TDA and TDB for 15 s, note the smallest and the largest readings during that period, and take the average of the two as the recorded reading. This technique could easily be used by unskilled operators. Statisticians refer to this technique as the "midrange" method of estimating the true reading. Taking the average is another technique for estimating the true reading. However, finding averages of repeated readings proved too tedious to be useful. The effectiveness of any approach for obtaining a good estimate of the true value depends on the distribution of the data points and other statistical considerations. Crow and Siddiqui (1967) discuss several different methods for obtaining good estimates for data points with various statistical distributions.

One of the tests made at the airport site provides a direct comparison of the "midrange" versus the "average" technique. Figure 1 shows the layout of the test. Markers were set at 200-ft increments along the airport taxiway, and two points were marked along an 890-ft line perpendicular to the taxiway that intersected the Coast and Geodetic Survey benchmark near the edge of the airport. The distances are indicated in figure 1, where points 1 through 8 are the locations where the readings were taken. Two sets of readings were taken at each point, except point number 8, where only one set was taken. One set of readings was taken while the operator was walking up the taxiway, and the second on the return trip. The results are shown in figures 2 and 3. The dashed lines connect averaged receiver readings while solid lines connect the midrange values. Also shown (displaced) are the gradients in microseconds per feet predicted for the airport site markers from Pierce et al.(1948).

As these figures show, the midrange approach does not yield as good a result as the average. The latter provides better repeatability, and the gradients derived from the average values come much closer to the gradients predicted for the system at the location of the measurements.

Finding the average would represent an important step forward in improving the performance of the manpack receivers. While it would be a tedious chore to obtain averages from the equipment in its present configuration, it would be relatively easy and inexpensive to incorporate an extra decade (or two) in the counter that drives the time-difference display and thus automatically to find the average of 10 (or 100) readings.

This test also demonstrates that the receiver can resolve incremental distances. Under conditions similar to those during the test and based on the average readings, the manpack receiver can easily resolve differential distances to better than 250 ft.

4. EQUIPMENT PERFORMANCE

The performance of the manpack receivers during both series of tests was best evaluated by examining the three principal features of the data obtained from them. These three features are: (1) the systematic differences between the observed and predicted readings; (2) the discrepancies between readings of the two receivers; and (3) the erratic behavior of the data from one receiver or the other on specific days.

Feature (1) is discussed in detail in later sections. In this later discussion, for both series of tests, it is assumed that the adverse contributions to the data from features (2) and (3) have been removed. Thus, in the later data analysis, which attempts to attribute the systematic discrepancies to either long-range or local effects, we assume we are working with the best possible data from the manpack receivers.

To eliminate the adverse contributions to the data from features (2) and (3) the following step was taken. The data from both series of tests were analyzed for obvious defects. The data from the second series of tests appeared to be much less variable than those of the first. Also, the discrepancy between the readings of the two receivers (units 2 and 5) for the second series appeared to be far less than the first, in which units 1 and 2 were used. The results of this cursory data analysis indicated that all data from the second series of tests should be used in the analysis of feature (1). However, the data from the first series needed culling.

To display the data and their characteristics from the first series of tests the mean value of the midrange readings for TDA and TDB for a given day on each receiver at each site was derived. This is plotted on the vertical scales in figures 4 through 14. On the abscissa, a line two standard deviations long, centered about zero, is drawn through the corresponding mean value. Solid lines identify unit 1; dashed lines, unit 2. The numbers beside the line show the date in May 1968 when the data were obtained. The numbers in parenthesis beside the line show the sample size for the particular receiver on that date.

From this display unsatisfactory receiver performance is easily observed. One indicator of unsatisfactory performance is when the readings on one day differ substantially from the readings taken at the same site by the same receiver on another day; indicative also is when the standard deviations are excessively large. Thus data from days of readings at a particular site are rejected if they have anomalously large standard deviations, or if their mean does not visibly cluster with the means for other days of readings at that site. These rejected data are included in figures 4 through 14 and are annotated by the letter "R", but these data are excluded from all subsequent analysis.

The Alabama tests of the first series produced a sufficient number of readings at each site that we believe that the rejection of the poor data leads to a better estimate of the overall mean value. However, in the Florida tests of the first series, there were fewer readings. Further, the spread in the day-to-day mean values was much greater in the Florida than in the Alabama tests. We thought it was desirable to select only the best data, but by rejecting apparently poor data, we may have seriously biased the outcome of the analysis of the Florida data.

A one-way analysis of variance was applied to the data behind figures 4 through 14 to test if the site-to-site differences in means were indeed systematic. For all cases, i. e., Florida TDA and TDB, and Alabama TDA and TDB, these differences were found to be systematic.

Thus, the equipment performed in a satisfactory manner for the most part. For the first series, where the two receivers showed large discrepancies and where the receivers behaved erratically, the data were eliminated from the analysis. For the second series, equipment performance was acceptable.

5. SYSTEMATIC DISCREPANCIES

Ideally, the differential loran technique should resolve the errors between the fixed and the mission receivers that are caused by temporal variations and offsets. That is, with temporal variations eliminated, the mission receiver's observed loran coordinates should locate a geographic point whose predicted loran coordinates are identical with those observed. In fact, however, the discrepancy between the observed and the predicted readings varied from site to site in both series of tests. These discrepancies, which will be analyzed in the following discussion, can be attributed to either long-range effects, i. e., the effects of perturbations integrated over

the total paths between the transmitters and receiver, or of localized perturbations. The two will be treated separately, but, as we shall see, the data are too scanty to determine whether the observed systematic discrepancies are caused by local or long-range effects.

5.1 Long-Range Effects

The time difference between the master and the slave stations is essentially a measured difference in phase, calibrated in microseconds, plus a constant. It can be described by the equation

$$TD = \nabla\varphi(\mu s) = k_1 (D_s - D_m) + k_2 (D_s - D_m) + k_3, \tag{1}$$

$$= (k_1 + k_2)(D_s - D_m) + k_3$$

where the quantities D_s and D_m are the geodesic distances (mi) between the slave transmitter and the receiver, and the master transmitter and the receiver respectively; k_1 and k_2 give the phase delay of the signals (μs/mi) as described by Johler, Kellar, and Walters (1956); and k_3 is the system constant that accounts for both the slave coding delay and the delay associated with the propagation time of the signal between the master and slave locations.

Equation (1) is generally used for calculating predicted time differences for loran systems. The coefficient k_1 equals the inverse of free-space velocity of light (5.36815 μs/mi) multiplied by a mean value for the refractive index of air (1.00034). Thus,

$$k_1 = 5.3700\ \mu s/mi.$$

Because the refractive index of air is so near unity, the error in time difference resulting from an error in the estimate of refractive index should not exceed $\pm$ 0.1 μs.

The coefficient k_2 is the secondary phase correction. For seawater, at distances of more than 300 mi from the transmitter, the secondary correction factor is independent of distance and has the value

$$k_2 = .00334 \mu s/mi.$$

For the overland propagation, this correction factor must be modified in a way that depends on the electrical properties of the ground. For example, for a homogeneous earth with a conductivity of 0.005 mhos/m and a relative dielectric constant of 15, k_2 becomes (Hefley, Linfield, Jones, 1955)

$$k_2 = .00727 \mu s/mi.$$

Discrepancies between observed and predicted values can appear whenever the assumptions implicit in (1) are violated. For example, an error in the estimate of the refractive index (which may vary slightly from day to day) can produce minor changes in the actual time-difference readings. For overland paths, two principal errors can arise that affect the coefficient k_2:

(a) An error caused by the mixed conductivities along the path. The appropriate correction factor may be different for the master and slave transmissions. The usual technique for handling this problem is to apply "Millington's Method" (Millington, 1949) to arrive at the appropriate mean correction factor for each path.

(b) Terrain irregularity effects. Recent calculations by Johler and Berry (1967) indicate that large phase discrepancies can occur in the vicinity of a terrain irregularity. Further, there is a (smaller) residual phase offset at large distances from the irregularity

that never recovers to the unperturbed value. These effects can be different for the master station and for each slave station propagation path.

There may also be an error in k_3 for several reasons:

(a) The system constant may be incorrect. This is particularly true of uncalibrated slave stations, such as the Dana slave, or the slave stations in Southeast Asia. Further, in such systems as loran-D, where the transmitters are transportable, the chain is not calibrated as a matter of course.

(b) When the conductivities for the two paths are different, a constant factor must be included in k_3. For seawater the secondary phase correction is given by

$$\nabla\varphi_{\text{secondary}}(\mu s) = -.29 + .00334D,$$

where D is the distance from a given transmitter. For a land conductivity of .005 mhos/m, for example, the secondary phase correction is given by

$$\nabla\varphi_{\text{secondary}}(\mu s) = +.64 + .00727D.$$

Thus, if the path to the receiver from the master were over seawater and the path from the slave were over land, with an effective conductivity of .005 mhos/m, a constant correction of 0.93 μs has to be accounted for. The reason such a factor does not appear in (1) is the assumption, implicit in the equation, that the earth is homogenous and that the two paths have identical secondary phase correction factors.

(c) Finally, and perhaps most important, is that constant systematic errors in the receiver can be accounted for by adjusting k_3. This is obvious from inspection of (1). Unfortunately, it is impossible to separate the constant systematic errors in the receiver from those in the loran chain.

In figure 15 the differences between the averages of the observed and the predicted TDA values (called the observed discrepancies) are shown by the open circles. TDA corresponds to the Cape Fear-Jupiter pair. The ordinate shows the difference between the observed and predicted values of TDA; the abscissa gives the difference in distance between the slave (D_s) and master (D_m) transmitters to each receiver site. The reason for choosing this particular quantity is that (D_s-D_m) appears naturally in the hyperbolic navigation equations. Further, since seawater conductivity parameters were used in determining the predicted TDA's, we would expect from theory that discrepancies proportional to (D_s- D_m) would appear because the paths are overland, where the secondary phase correction differs substantially from the correction appropriate for seawater. This is discussed further below.

Figure 15 shows clearly that a systematic discrepancy does appear, but it is not solely due to choosing the wrong conductivity correction factor in the simple theory. The simple theory (based on uniform conductivity and a smooth earth) indicates that systematic discrepancies caused by this effect alone would be zero when (D_s-D_m) = 0. Figure 15 shows that the discrepancy is zero near (D_s-D_m) $\approx$ 110 mi. Further, the slope of the best straight line drawn through the points in figure 15 is much too large to be the result of an error in the estimate of the conductivity. The slope based on the measured points is about .026/μs/mi, while the slope predicted for an average conductivity of .005 mhos/m is

only about .004 μs/mi. A conductivity of .005 mhos/m is generally accepted as a reasonable value for the region over which the signals propagated. However, even if the conductivity were very small (.0001 mhos/m), the appropriate correction would be insufficient to account for the large slope measured. Even "Millington's Method", which in essence predicts a correction factor based on a weighted mean conductivity for each path, could therefore not properly account for the large slope observed.

Figure 16 shows different but analogous properties in the behavior of TDB for the Alabama sites. The TDA's and TDB's for the Florida sites also show analogous properties and are shown in figures 17 and 18. Figures 19 through 22 show similar behavior in the data for the second series of tests.

If the discrepancies between observed and predicted time differences are due to long-range effects, it is reasonable to assume that most of them can be accounted for by an equation similar in form to (1). For calculation purposes, (1) can be written as

$$TD = k_0 (D_s - D_m) + k_3, \tag{2}$$

where $k_0 = k_1 + k_2 = 5.37334$ is the value used in the loran navigation equation, and k_3 is the system constant. For our discussion here, however, it is best to recast it into the form

$$TD = K_1 D_s - K_2 D_m + K_3, \tag{3}$$

where $K_1 = K_2 = k_0$, and $K_3 = k_3$. The discrepancy between the observed and predicted time differences (ΔTD) can be accounted for by the equations

$$\Delta TDA = \epsilon_1 D_{sA} - \epsilon_2 D_m + \epsilon_4,$$

and

$$\Delta TDB = \epsilon_3 D_{sB} - \epsilon_2 D_m + \epsilon_5, \tag{4}$$

where the ϵ's account for the errors in the K's for the reasons enumerated above. Two equations are required, one for ΔTDA and one for ΔTDB, since the propagation paths from the respective slave stations are different. Thus, the discrepancy in K_1 and K_3 for the two time-difference readings may well be different. The two time-difference equations, however, do have K_2 in common.

Using the measured values of ΔTDA and ΔTDB at the sites used in the experiment, our objective is to determine the values of the ϵ's (and their standard deviations) that give the best fit to the data. Whatever residuals that remain after the best fit is obtained will depend to some extent on the statistical processing of the data to determine the ϵ's on their confidence intervals, but the residuals can be assumed to be due principally to the phase perturbations resulting from local terrain effects and to other effects that may enter in a nonlinear way. Also, since in some instances the data recording sites were offset from the benchmarks, offset errors will also affect the residuals. At the Taylor site for example, the offset was particularly large. No attempt was made to correct for the offsets, since no information on offset distance or direction was available.

5.2 Statistical Considerations

To obtain the least square estimate of the ϵ's, we used the Gauss-Markoff least squares theorem (David and Neyman, 1938) to minimize the quantity

$$S \equiv \sum_{i=1}^{N} w_{Ai}(\Delta TDA_i - \epsilon_1 D_{sAi} + \epsilon_2 D_{mi} - \epsilon_4)^2 + w_{Bi}(\Delta TDB_i - \epsilon_3 D_{sBi} + \epsilon_2 D_{mi} - \epsilon_5)^2 \quad (5)$$

with respect to the ϵ's. The quantities ΔTDA_i and ΔTDB_i are the differences between the observed and predicted TDA's and TDB's at the i sites ($i = 1, \ldots, N$); w_{Ai} and w_{Bi} are the weights to be assigned to ΔTDA_i and ΔTDB_i, respectively; D_{sAi}, D_{sBi}, and D_{mi} are the distances from the i^{th} site to slave A, slave B, and the master transmitters, respectively; the ϵ's are the quantities, to be determined, that give the best linear fit to the data; and N is the number of sites at which measurements were taken.

The standard deviations of the ϵ's can be estimated through an estimate of the variances of the ϵ's. The latter estimate is given by

$$(\text{estimate of variance of } \epsilon_j) = \frac{S_o}{2N-5} \sum_{i-1}^{N} \frac{\delta^2_{Aji}}{w_{Ai}} + \frac{\delta^2_{Bji}}{w_{Bi}}, \quad j = 1, \ldots, 5 \qquad (6)$$

where S_o is the minimum value of S obtained by substituting the estimates of the ϵ's determined by the least squares minimization procedure into (5), and δ_{Aji} and δ_{Bji} are the coefficients of ΔTDA_i and ΔTDB_i, respectively, in the expression for ϵ_j expressed in terms of ΔTDA_i and ΔTDB_i. The equations for the minimization conditions and (6) are easily expressed in matrix form for easier computation. Implicit in (5) is the assumption that ΔTDA_i and ΔTDB_i are mutually independent. This may well not be the case, but, if not, the solution is much more complicated but not necessarily much better, for we would have to approximate covariances (or correlations) for ΔTDA_i and ΔTDB_i.

Another question in treating the data statistically is how best to assign the weights w_{Ai} and w_{Bi}. If all data contributing to the determination of the mean ΔTDA_i and ΔTDB_i were independently distributed, an appropriate weight for each ΔTDA_i and ΔTDB_i would be the inverse of the variance of the mean, which is given by

$$w_{AI} = \frac{n_i}{S^2_{Ai}}$$

and

$$w_{Bi} = \frac{m_i}{S^2_{Bi}} ,$$

where S^2_{Ai} and S^2_{Bi} are the sample variances calculated from the n_i and m_i observations at the i^{th} site. On the other hand, if the over-riding errors can be ascribed to random systematic site errors, it would be more appropriate to weight all the ΔTDA_i and ΔTDB_i equally, i. e., $w_{Ai} = w_{Bi} = 1$. Although the latter probably comes closer to the actual experimental situation than the former, both weighting techniques were applied with similar results. For both cases, data points that appeared to reflect obvious equipment difficulties, as described in section 4, were not included in the analysis.

Table 3 shows the comparison for each of the 11 sites used in the first series of tests of the observed and calculated ΔTDA's and ΔTDB's obtained from the least squares fit calculated for both types of weighting factors discussed above. The points predicted by this technique are shown in figures 15 through 18 by crosses for weights $w_i = n_i/S_i^2$ and by open squares where the weights are all equal to unity. The analogous comparisons for the 13 sites from the second series of tests are given in table 4 and in figures 19 through 22. Table 5 gives the ϵ's and the estimates of their standard deviations (σ) for the first series of tests; and table 6 gives the values derived from the second series. The values derived for ϵ_1, ϵ_2, and ϵ_3 should be compared with .0039, which is the value we would expect if the errors in the predicted values were due solely to a secondary phase correction for

Table 3. May 1968 Tests.

Site	Observed ΔTDA	Calculated Least Squares Fit ΔTDA				Observed ΔTDB	Calculated Least Squares Fit ΔTDB			
		$w_{Ai} = \frac{n_i}{\sigma^2_{A_i}}$		$w_{Ai} = 1$			$w_{Bi} = \frac{m_i}{\sigma^2_{B_i}}$		$w_{Bi} = 1$	
		ΔTDA	Residual	ΔTDA	Residual		ΔTDB	Residual	ΔTDB	Residual
Delta	-.31	- .38	.07	- .31	.00	-1.35	-1.22	-.12	-1.25	-.10
Taylor	-.15	- .03	-.12	- .01	-.14	-0.79	-1.06	.27	-1.22	-.43
TT-1	.20	+ .17	.03	.20	.00	-1.73	-1.39	-.34	-1.65	-.07
Airport	-.05	- .06	-.07	- .02	-.03	-1.52	-1.39	-.13	-1.54	.02
Horn	-.54	- .89	.25	- .64	.10	-1.84	-1.96	.12	-1.82	-.02
Able	+.05	- .04	.09	- .01	.06	-1.54	-1.13	-.41	-1.29	-.24
Goose	-2.04	-2.05	.01	-2.04	.00	.56	.61	-.05	0.59	-.03
West	-1.76	-1.95	.19	-1.85	.09	.28	.27	.01	0.25	.03
Burnt	-1.81	-1.75	-.06	-1.68	-.13	.35	.50	-.15	0.45	-.10
Southport	-1.57	-1.74	.17	-1.71	.14	.62	.60	.02	0.54	.08
Park	-2.03	-2.00	-.03	-1.93	-.10	.38	.36	.01	0.35	.02

Table 4. May 1969 Tests.

Station	Observed ΔTDA	Calculated Least Squares Fit ΔTDA				Observed ΔTDB	Calculated Least Squares Fit ΔTDB			
		$w_{Ai} = \frac{n_i}{\sigma^2_{A_i}}$		$w_{Ai} = 1$			$w_{Bi} = \frac{m_i}{\sigma^2_{B_i}}$		$w_{Bi} = 1$	
		ΔTDA	Residual	ΔTDA	Residual		ΔTDB	Residual	ΔTDB	Residual
Mead	.19	.12	.07	-.15	.04	-1.50	-1.60	.10	-1.56	.06
Piedmont	.39	.44	-.05	.49	-.10	-1.22	-1.34	.12	-1.28	.06
Airport	.08	.02	.06	.04	.04	-1.47	-1.47	.00	-1.42	-.05
Delta	-.15	-.15	.00	-.13	-.02	-1.22	-1.28	.06	-1.21	.01
TT-1	.24	.17	-.07	.17	.08	-1.68	-1.55	-.13	-1.50	-.18
Bynam	-.03	.02	-.05	.04	-.06	-1.46	-1.60	.14	-1.55	.09
Able	.21	.02	.19	.05	.16	-1.62	-1.32	-.30	-1.26	-.36
Taylor	-.08	.02	-.10	.05	-.13	- .81	-1.28	.47	-1.21	.40
Goose	-1.83	-1.82	-.01	-1.83	.00	.65	.77	-.12	.68	-.03
Park	-1.78	-1.70	-.08	-1.72	-.06	.40	.51	-.11	.44	-.04
West	-1.64	-1.62	-.02	-1.64	.00	.47	.37	.10	.33	.14
Burnt	-1.56	-1.57	.01	-1.55	-.01	.21	.39	-.18	.43	-.22
Southport	-1.50	-1.60	.10	-1.57	.07	.67	.48	.19	.51	.16

Table 5. May 1968 Tests.*

	ALABAMA				FLORIDA			
	$w_{A_i} = \frac{n_i}{\sigma^2_{A_i}}$; $w_{B_i} = \frac{m_i}{\sigma^2_{B_i}}$		$w_{A_i} = w_{B_i} = 1$		$w_{A_i} = \frac{n_i}{\sigma^2_{A_i}}$; $w_{B_i} = \frac{m_i}{\sigma^2_{B_i}}$		$w_{A_i} = w_{B_i} = 1$	
	ϵ	σ	ϵ	σ	ϵ	σ	ϵ	σ
ϵ_1	.0387†	.0154	.0338†	.0112	.0119†	.0031	.0157†	.0050
ϵ_2	.0315†	.0101	.0257†	.0069	.0257†	.0070	.0221†	.0080
ϵ_3	.0062	.0119	.0063	.0095	.0022	.0065	.0038	.0053
ϵ_4	-7.5861	8.3245	-7.370	6.2918	6.9175	3.21	4.3430	3.9504
ϵ_5	15.7649	7.4480	7.4588	4.8989	12.5869†	4.75	9.5822	5.8626

*
The ϵ's (defined on p. 13) account for the errors in the K's for the reasons enumerated on pp. 8-10. Again, the reader is reminded that the ϵ's are statistical in nature and the quality of the data as well as the small number of sites used in deriving the ϵ's do not allow high statistical confidence in the analytical results (see p. 24).

†
Only these ϵ's are significantly different from) as judged by a deviation of 2σ or more.

seawater conductivity being used in place of the secondary phase correction appropriate for overland propagation where the ground conductivity was .005 mhos/m. The values of ϵ_1 and ϵ_2 are consistently much larger than this by more than 2σ. The values for ϵ_3 are relatively small and, because of their large standard deviation, might be interpreted as consistent with a secondary phase correction factor for a homogeneous earth.

The values of the ϵ's in the second test series appear to differ substantially from those in the first, but the estimates of standard deviations for the two test series are such that, statistically, they can be taken to represent two different estimates of the same values. The values of ϵ_4 and ϵ_5 appear to be particularly sensitive to the set of data points, as reflected in the large estimates of standard deviations for these two parameters.

Even allowing for the complexity of the paths, which contain mixed conductivities and considerable terrain irregularity, the values of ϵ_1 and ϵ_2 are surprisingly high. On the basis of current theory, this error in the secondary phase correction factor can be accounted for only by much mountainous terrain with poor conductivity over which the radio wave propagates. Yet, the derived values of ϵ_3 are small enough to be consistent with homogeneous, smooth-earth theory, even though the propagation paths from the Dana slave transmitter to both the Alabama and Florida areas cross the Appalachian Mountains.

It is interesting to compare these results with the maps of effective conductivity at 10 kHz prepared by the Georesearch Laboratory of the Westinghouse Electric Corporation (Morgan and Maxwell, 1965). These small-scale maps show expected average conductivities over large areas of the USA for 10 kHz. Conductivities at 10 kHz cannot be directly applied to 100 kHz signals, but they do provide guidelines for

Table 6. May 1969 Tests*.

	ALABAMA				FLORIDA			
	$w_{A_i} = \frac{n_i}{\sigma^2_{A_i}}$; $w_{B_i} = \frac{m_i}{\sigma^2_A}$		$w_{A_i} = w_{B_i} = 1$		$w_{A_i} = \frac{n_i}{\sigma^2_{A_i}}$; $w_{B_i} = \frac{m_i}{\sigma^2_{B_i}}$		$w_{A_i} = w_{B_i} = 1$	
	ϵ	σ	ϵ	σ	ϵ	σ	ϵ	σ
ϵ_1	.0177†	.0037	.0186†	.0075	.0091	.0059	.0106	.0061
ϵ_2	.0149†	.0046	.0161†	.0064	.0088	.0098	.0143	.0097
ϵ_3	.0052	.0044	.0056	.0068	.0133	.0114	.0080	.0065
ϵ_4	-3.1917	2.8610	-3.1293	4.3694	-.6681	4.5108	1.6302	4.8281
ϵ_5	3.0287	2.8228	3.4638	4.3036	-3.7836	9.1310	2.6332	7.1651

* The ϵ's (defined on p. 13) account for the errors in the K's for the reasons enumerated on pp. 8-10. Again, the reader is reminded that the ϵ's are statistical in nature and the quality of the data as well as the small number of sites used in deriving the ϵ's do not allow high statistical confidence in the analytical results (see p. 24).

† Only these ϵ's are significantly different from) as judged by a deviation of 2σ or more.

what can be expected at 100 kHz. They show that for the Alabama tests the signals from the Dana slave propagate largely over terrain of relatively high conductivity (3×10^{-2} mhos/m), while the path from the Jupiter slave is over terrain of moderate conductivity (3×10^{-3} mhos/m). The path to the master over the Appalachian Mountains involves relatively low, but highly variable ($\pm$ 1 decade), effective conductivities. The effect of this wide variation in conductivity between paths seems to correlate well with the trends of the values of ϵ_1 through ϵ_3 for the Alabama test, even though detailed predictions based on these conductivities (based on the simple theory) do not agree with the derived values of the ϵ's.

If the observed discrepancies are the result of long-range effects, the waves must be slowed by a mechanism not accounted for in present ground-wave propagation theories in order to account for the large correction factors. We know that highly inductive surfaces, such as corrugated surfaces (Wait, 1957), uniformly rough surfaces (Wait, 1959), or stratified ground (Wait, 1956), can all produce slowed waves. It is not difficult to visualize geophysical configurations that these models approximate in the frequency range appropriate to loran propagation. Unfortunately, the quality of the data and the small number of sites used in deriving the values for the ϵ's do not allow high statistical confidence in the results of the analysis.

5.3 Local Effects

Such geophysical factors as irregular terrain, conductivity inhomogeneities, and ground stratification that lead to a re-examination of the secondary phase correction factors for long-range propagation effects also imply that we should expect local anomalies in the time-difference readings. This "grid warp" in regions of irregular terrain

was first observed during evaluation tests of the CYTAC system (Linfield, Doherty, and Hefley, 1957). The question arises, then, whether the observed systematic discrepancies are due to local rather than long-range effects. Figure 23 shows the correction factors for the May 1968 tests in Alabama. This figure is based on the calculated least-squares fit for ΔTDA and ΔTDB with unity weights and well known contouring techniques. That is, by assuming the contours to be represented by linear functions of latitude and longitude we used the methods of section 5.2 to solve the equations

$$\Delta TDA = a_1 L_a - a_2 L_o + a_4$$

and (7)

$$\Delta TDB = a_3 L_a - A_2 L_o + a_5 .$$

In equations (7), L_a, L_o are the latitude and longitude, respectively, and the a_i (i = 1, 2, 3, 4, 5) were determined from a least-squares fit to the data. With the a_i determined for the area, the contouring techniques were employed to plot the lines of constant ΔTDA and ΔTDB at equal intervals. The locations of the Alabama test sites are shown for reference and to indicate the scatter of the data points on which the analysis is based. A user of figure 23 can predict loran coordinates TDA and TDB for a given latitude and longitude within the area and then add the interpolated values of ΔTDA and ΔTDB to obtain a good estimate of what he can expect to observe at that geographic location.

For these data, the rms error of the uncorrected discrepancies is 0.24 μs for TDA and 0.34 μs for TDB. Equations (7) above - or (4) - were used to help reduce the systematic discrepancies. This analysis yields a residual rms error of 0.08 μs for TDA and 0.21 μs for TDB with unity weight. Similar comparisons of uncorrected rms errors with

residual rms errors for the remaining data of both series show a reduction by about a factor of 2 in the rms errors when corrections are applied.

6. DISCUSSION AND RECOMMENDATIONS

This evaluation of the loran-C manpack tests centers on a discussion of equipment performance and signal propagation, and how they are related to discrepancies between observed and predicted loran fixes. Since the discrepancy appears to change systematically from site to site, corrections to the propagation factors are needed to improve loran position determination, even when the differential technique is used.

The equipment in the first series of tests performed erratically on some occasions, and when the two receivers were operated side by side discrepancies appeared between readings. The equipment operation in the second series was much more stable, and discrepancies between the two receivers were smaller. Such discrepancies between readings may be partly the result of using midrange readings rather than averages. The airport test indicated that time-averaged data from normally operating receivers have better repeatability than the data derived by a midrange technique. Since the average is a better statistical estimate of the true reading than the midrange value, one recommendation for improving the receiver performance is to include an automatic averaging capability for either 10 or 100 consecutive readings. Also, since both receivers were observed to perform erratically, it might improve the differential loran technique if additional receivers for the tests are used, so that receivers at all the sites can be compared. This procedure would make it easier to identify erratic performance in any given receiver

and thus insure greater reliability in site readings for systematic discrepancy evaluation.

The systematic variation in discrepancies from site to site appears to be the result of a combination of propagation effects, constant systematic errors in either the loran chain or the equipment, and imperfect techniques. In the analysis of the systematic errors, separate equipment errors from errors and unknowns of the loran grid cannot be separated. The tests by NASL were run primarily to examine the performance of the manpack receivers. The systematic discrepancies being discussed here are more a matter of understanding and correcting for errors that result from unpredicted propagation effects, which represent the ultimate limitation on loran accuracies.

The analysis technique, based on known statistical methods, shows that the errors attributed either to long-range propagation effects or to local effects, plus the system error (which includes errors in the receivers), can be estimated within a statistical confidence interval. From statistical theory, we know that the confidence in the estimates can be increased by increasing the number of test sites. In the first series of tests, six sites were used in Alabama and five in Florida; in the second, eight sites were used in Alabama and five in Florida. For long-range effects, a critical quantity in the analysis - see (6) - is $1/(2N-5)$, where N is the number of sites. Increasing N from 6 to 12 almost trebles the statistical reliability, and N = 20 could increase it by a factor of 5. This would be useful for determining the ϵ_i's with greater reliability. With these reliable estimates of the ϵ's, the ΔTD's in (4) can be added to the TD's in (3), and the new equation can then be used to predict readings at other sites in the area, i. e., to provide a loran-C grid for an area where a large percentage of the systematic errors have been eliminated. A greater number of data points are also

required to determine whether the discrepancies are local in origin rather than the result of long-range phenomena. A good rule of thumb is that there should be roughly three times the number of data points as there are parameters to be fitted. Thus, to evaluate grid warp due to local effects, a much larger number of data points are required. With information currently available, however, it is virtually impossible to decide whether the observed discrepancies are due to local or long-range effects, or to a combination of both.

Although the quality of the data and the small number of sites used in deriving the correction factors do not allow high statistical confidence in the analytical results, the systematic discrepancies are obvious, and the least squares fit technique apparently is successful in reducing the rms discrepancies by about a factor of 2. Further, figures 15 through 22 show that most of the systematic variations from site to site can be removed by evaluating the ϵ's by fitting the observed data to the model. Thus, if we add (3) and (4) and use the resulting equation to calculate "new predicted" values, this new equation can be used to predict the time difference reading at other sites in the area, with the expectation that systematic discrepancies will be reduced.

As an example, in the second series, each of the Alabama test sites was removed from the least squares analysis one by one, and "new predicted" values were computed from the remaining data for the removed sites. The difference between the observed and the "new predicted" values is given in table 7 along with the ϵ's computed on the basis of the measurements reduced by data taken at the location being "predicted" for. Compare the difference between the observed TDA and the "new predicted" TDA, i.e., the "new predicted" ΔTDA, in table 7 with the observed ΔTDA in table 4 for corresponding stations. This comparison shows "new predicted" ΔTDA values to be smaller

Table 7. May 1969 Tests.*

		$w_{A_i} = n_i/\sigma^2_{A_i}$			$w_{B_i} = m_i/\sigma^2_{B_i}$	
Station Eliminated	TDA Observed	"New Predicted" TDA	"New Predicted" ΔTDA	TDB Observed	"New Predicted" TDB	"New Predicted" ΔTDB
Mead	14,308.40	14,308.30	0.10	68,389.95	68,389.84	0.11
Piedmont	14,422.33	422.25	0.08	445.40	444.91	+0.49
Airport	14,281.71	281.65	0.06	468.56	468.56	0.00
Delta	14,236.51	236.51	0.00	589.56	589.31	0.25
TT-1	14,318.18	318.06	0.12	408.72	408.96	-0.24
Bynum	14,275.78	275.78	0.00	412.51	412.30	0.21
Able	14,289.87	289.69	0.18	537.02	557.37	-0.35
Taylor	14,292.61	292.78	-0.17	557.54	557.08	0.46

	ϵ_1	σ	ϵ_2	σ	ϵ_3	σ	ϵ_4	σ	ϵ_5	σ
Mead	.0169	.0042	.0166	.0055	.0057	.0048	-1.9714	3.6139	3.6301	3.1854
Piedmont	.0150	.0069	.0044	.0104	.0156	.0072	-6.4051	3.0393	-6.3151	7.3442
Airport	.0175	.0040	.0153	.0053	.0052	.0049	-2.8972	3.2567	3.2455	3.2072
Delta	.0174	.0063	.0130	.0053	-.0011	.0081	-3.8807	4.1564	4.8728	3.5573
TT-1	.0172	.0033	.0122	.0045	.0021	.0042	-4.1660	2.7756	3.2604	2.5587
Bynum	.0178	.0040	.0155	.0055	.0052	.0048	-3.0154	3.3719	3.2839	3.2016
Able	.0181	.0033	.0155	.0042	.0063	.0040	-3.1368	2.5603	2.8876	2.5072
Taylor	.0170	.0036	.0151	.0046	.0047	.0043	-2.6467	2.858	3.3720	2.7418

*See Table 5 or 6 for footnote.

in absolute value than the corresponding observed ΔTDA at seven out of eight locations. A similar comparison for ΔTDB values of the two tables shows the "new predicted" ΔTDB to be considerably less, in absolute value, in all cases than the observed ΔTDB. Results of computing rms values for the observed ΔTDA and ΔTDB are 0.17 and 0.26, respectively, while the corresponding "new predicted" ΔTDA and ΔTDB have rms values of .11 and .24. One would not expect much improvement, if any, in rms values because of the relatively small number of sites used in the experiment. But the method of correcting an area by calculating "new predicted" time differences based upon the ϵ's for a given set of sites appears to be a good method of reducing the systematic discrepancy.

The analysis here is far from conclusive but appears to have promise. We recommend that in possible future experiments to determine long-range and/or local loran propagation effects in a given area of relatively small diameter measurements be made at a substantial number of first order survey sites with high quality loran receivers. With the added confidence of adequate measurments at 20 or more sites within a locale, it should be possible by the prescribed method of reducing systematic discrepancies to isolate long-range from local effects and provide an effective means for "calibrating" a given locale for determining precise loran position.

7. ACKNOWLEDGEMENTS

The authors wish to thank Dr. Edwin L. Crow for his assistance in the statistical approach in this paper and Mr. Robert H. Doherty for his critical and helpful review of an earlier manuscript. Mr. Doherty pointed out that it would be possible to correct for the gross loran discrepancies by use of the classical theory.

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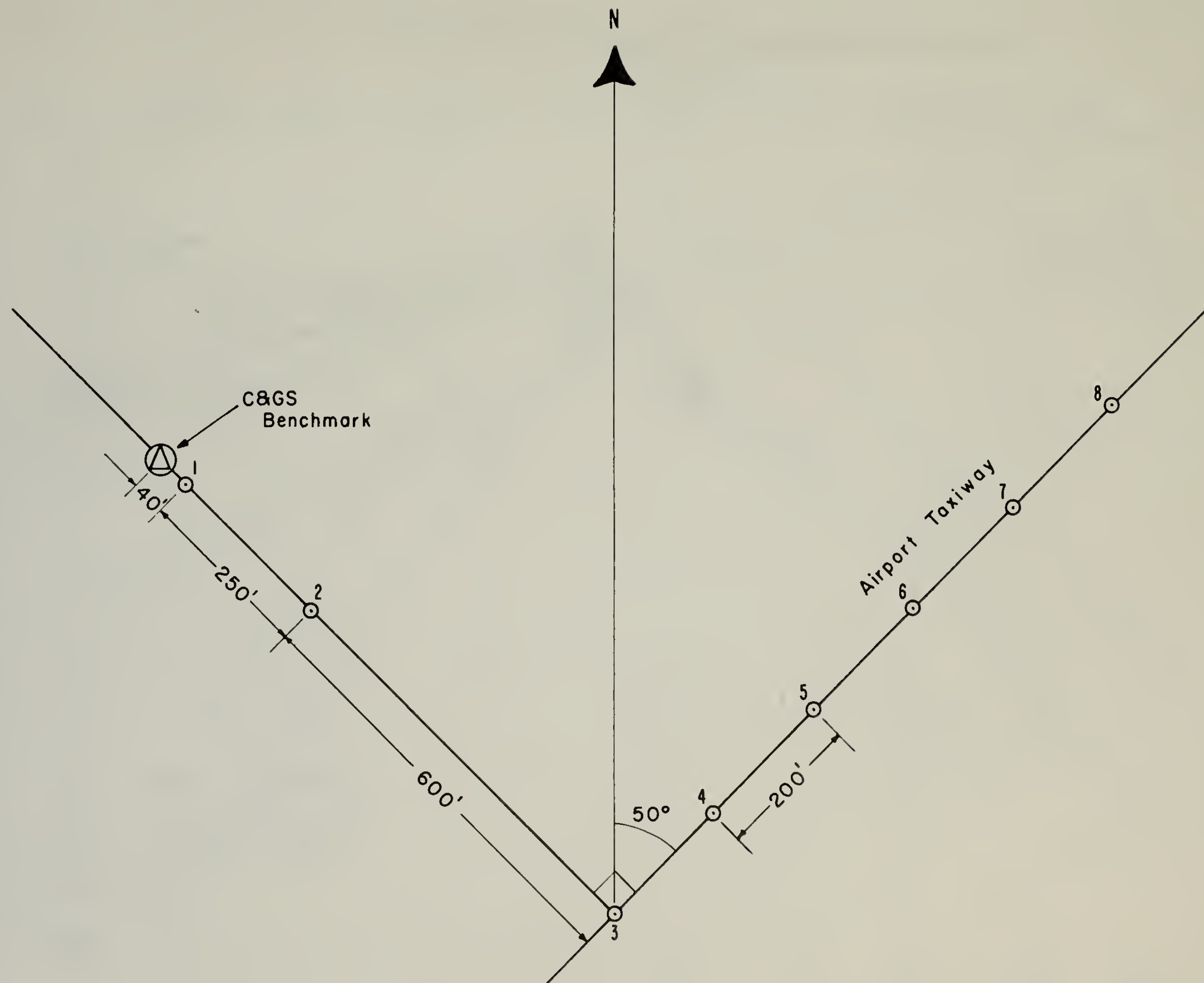

Figure 1. Layout of airport test site.

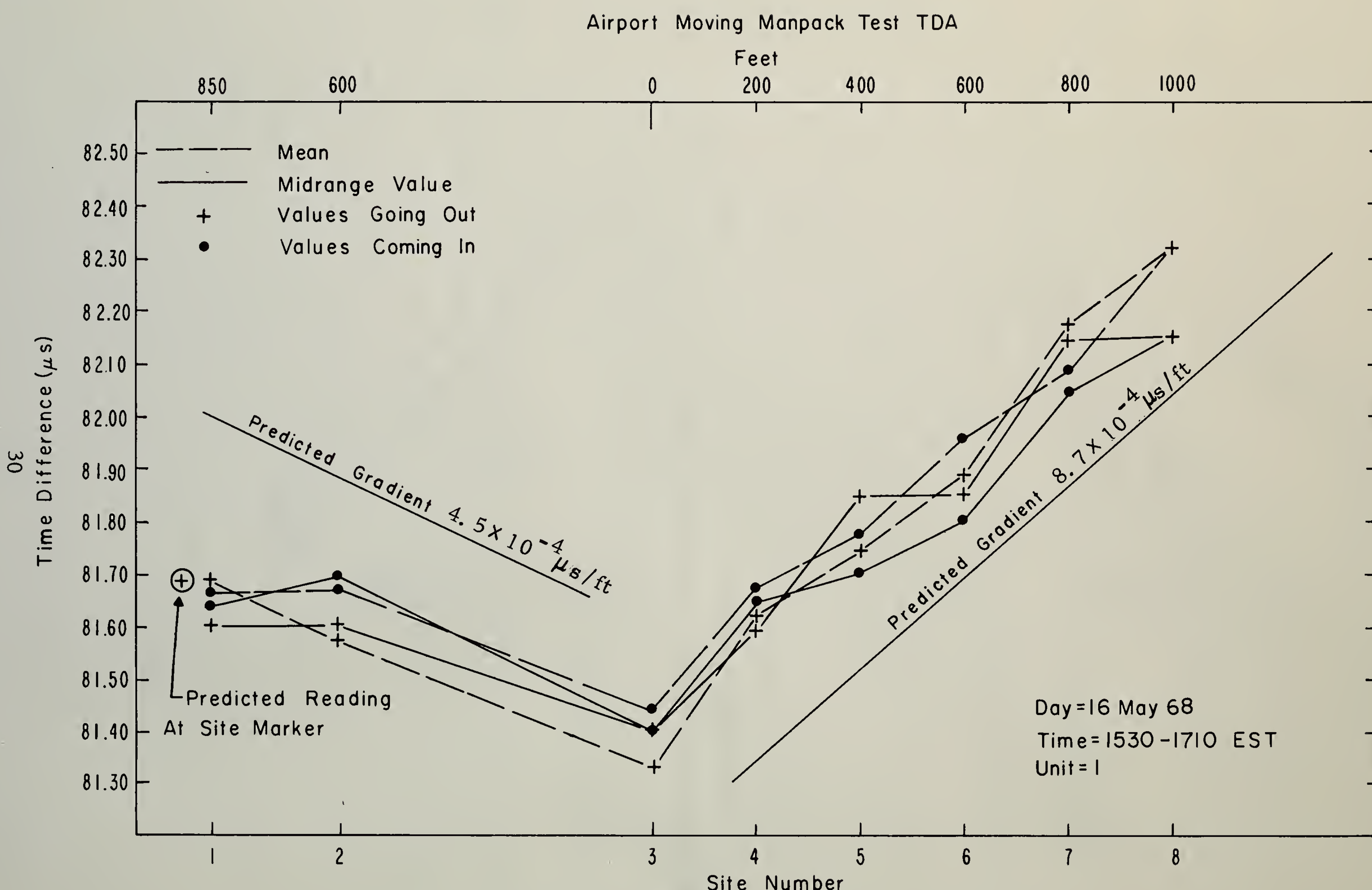

Figure 2. Airport moving manpack test TDA.

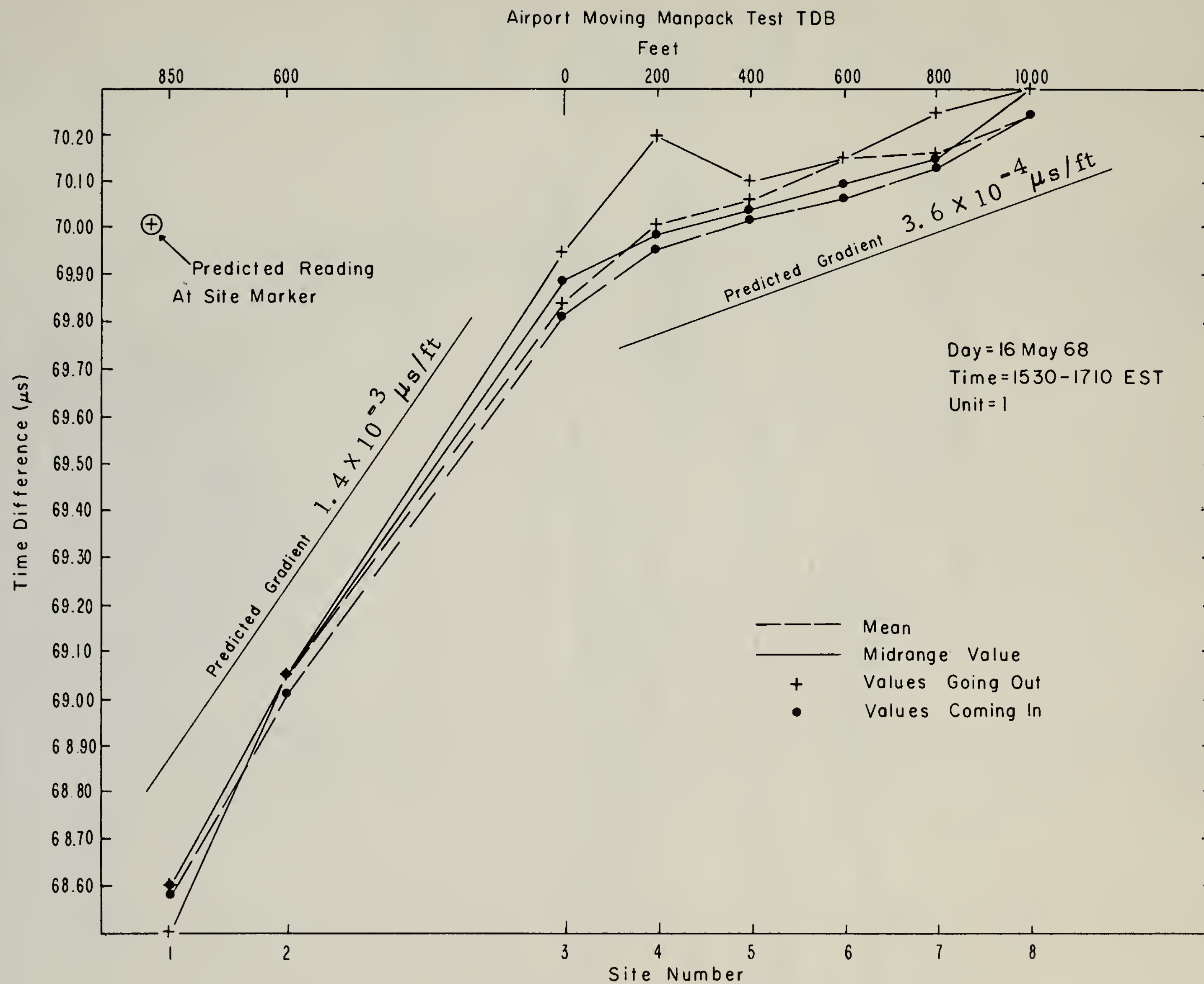

Figure 3. Airport moving manpack test TDB.

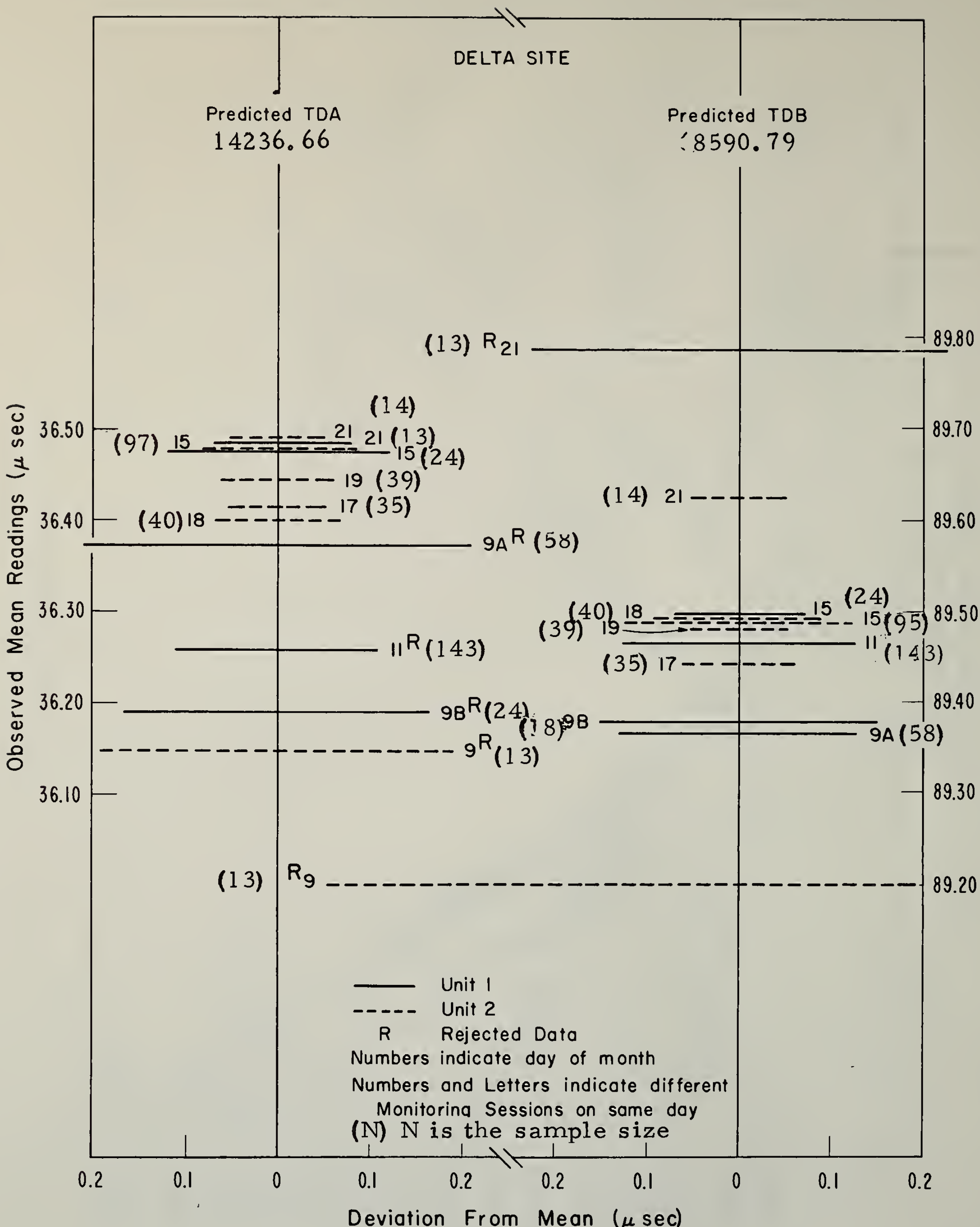

Figure 4. Data dispersion at Delta site 1968.

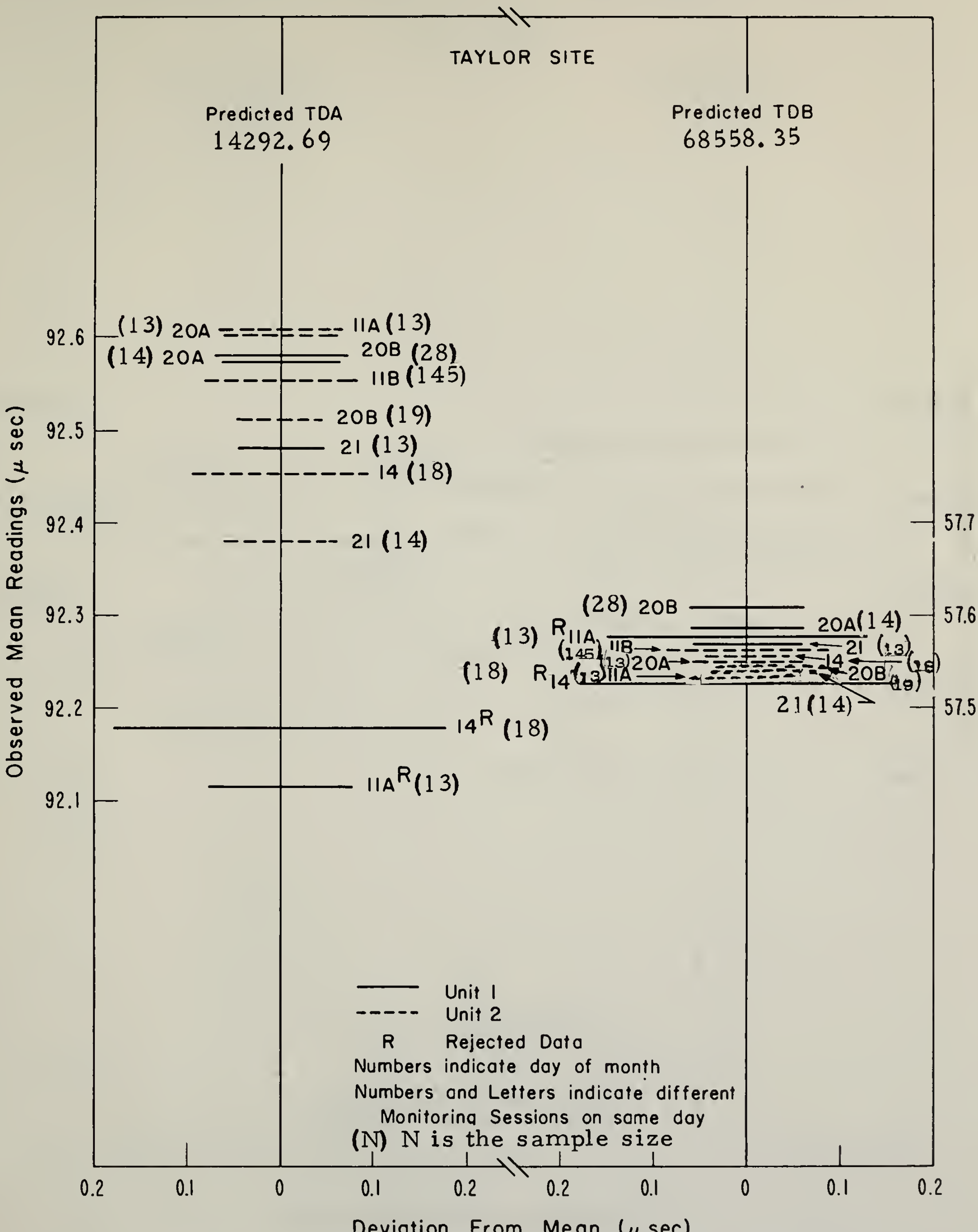

Figure 5. Data dispersion at Taylor site 1968.

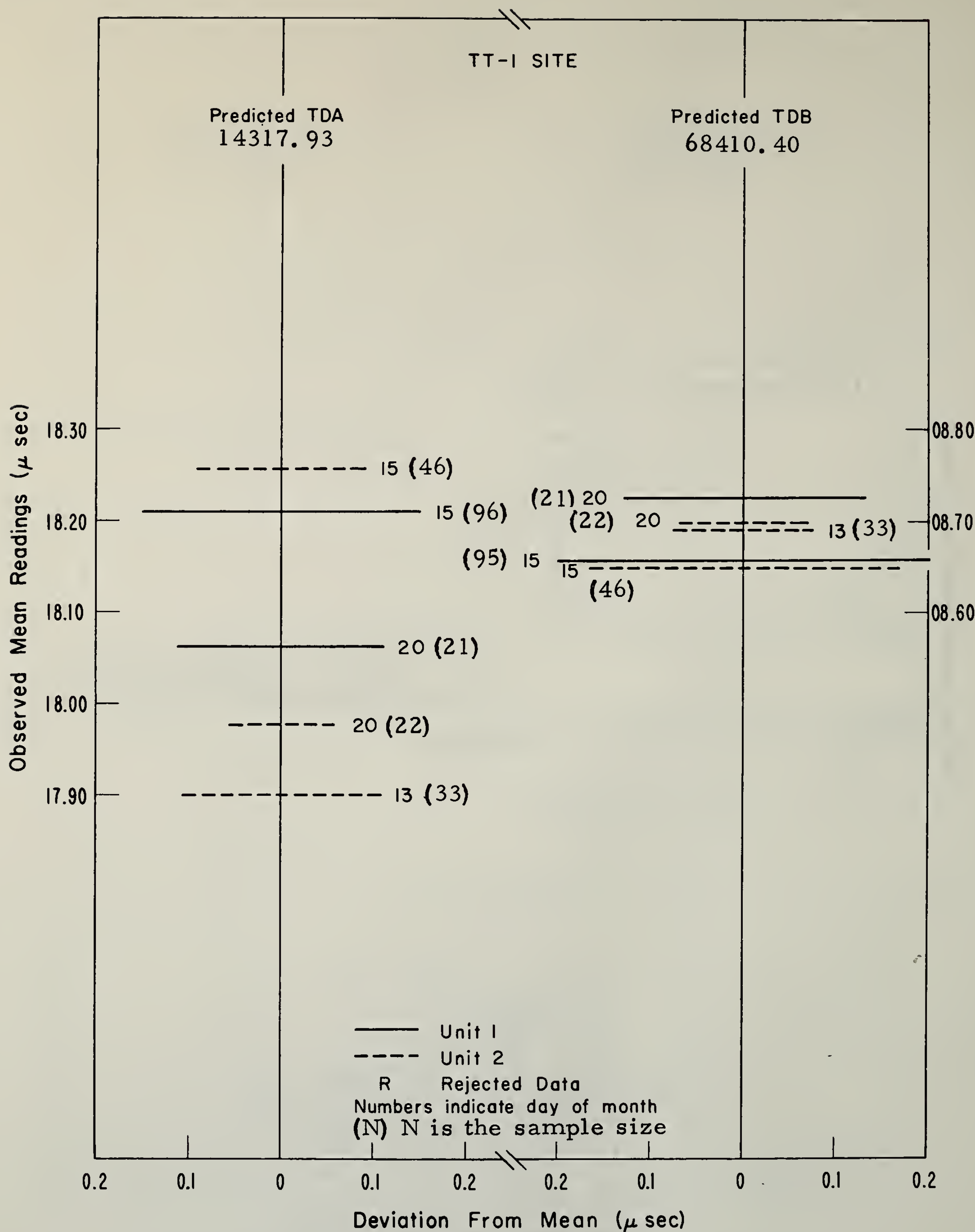

Figure 6. Data dispersion at TT-1 site 1968.

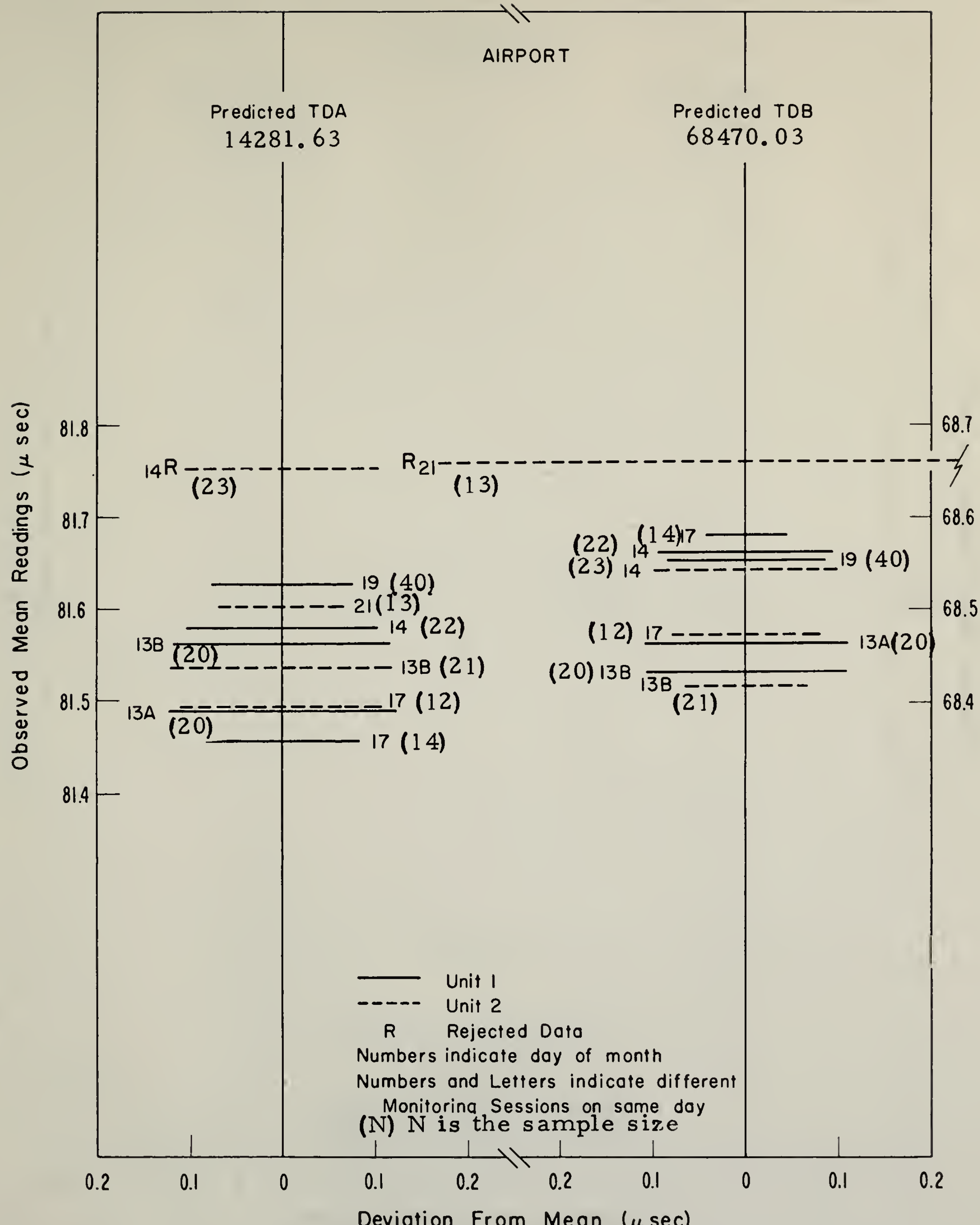

Figure 7. Data dispersion at Airport site 1968.

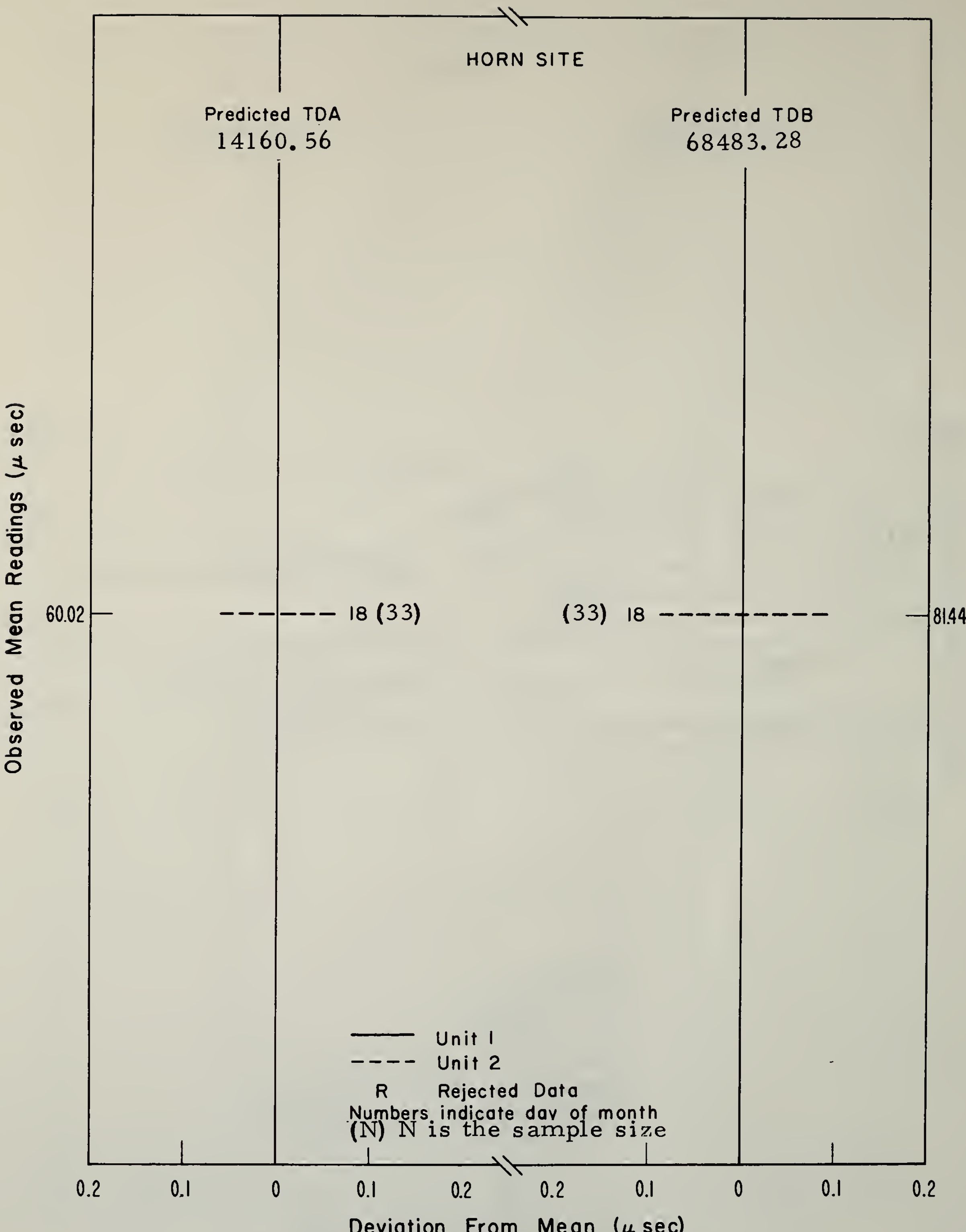

Figure 8. Data dispersion at Horn site 1968.

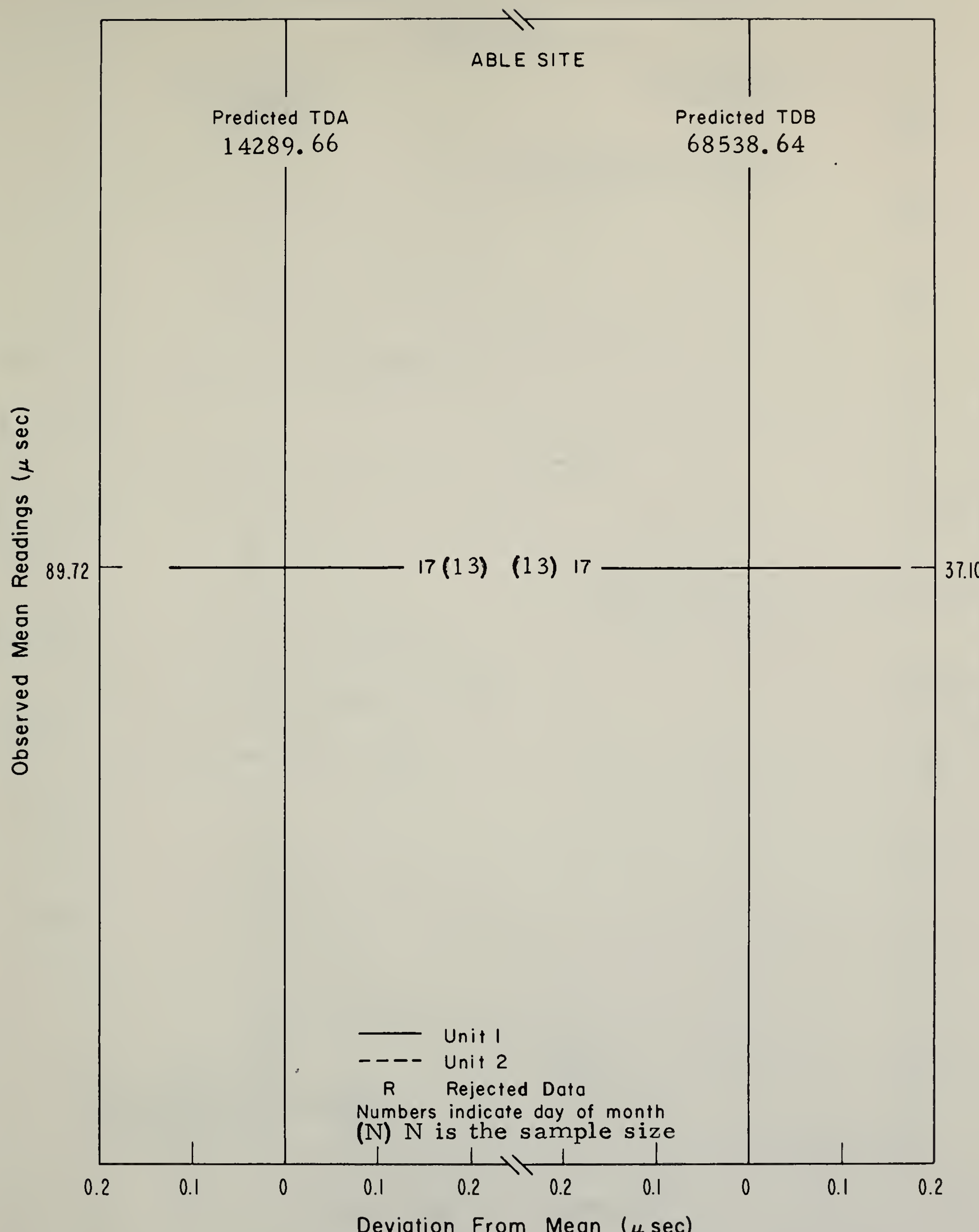

Figure 9. Data dispersion at Able site 1968.

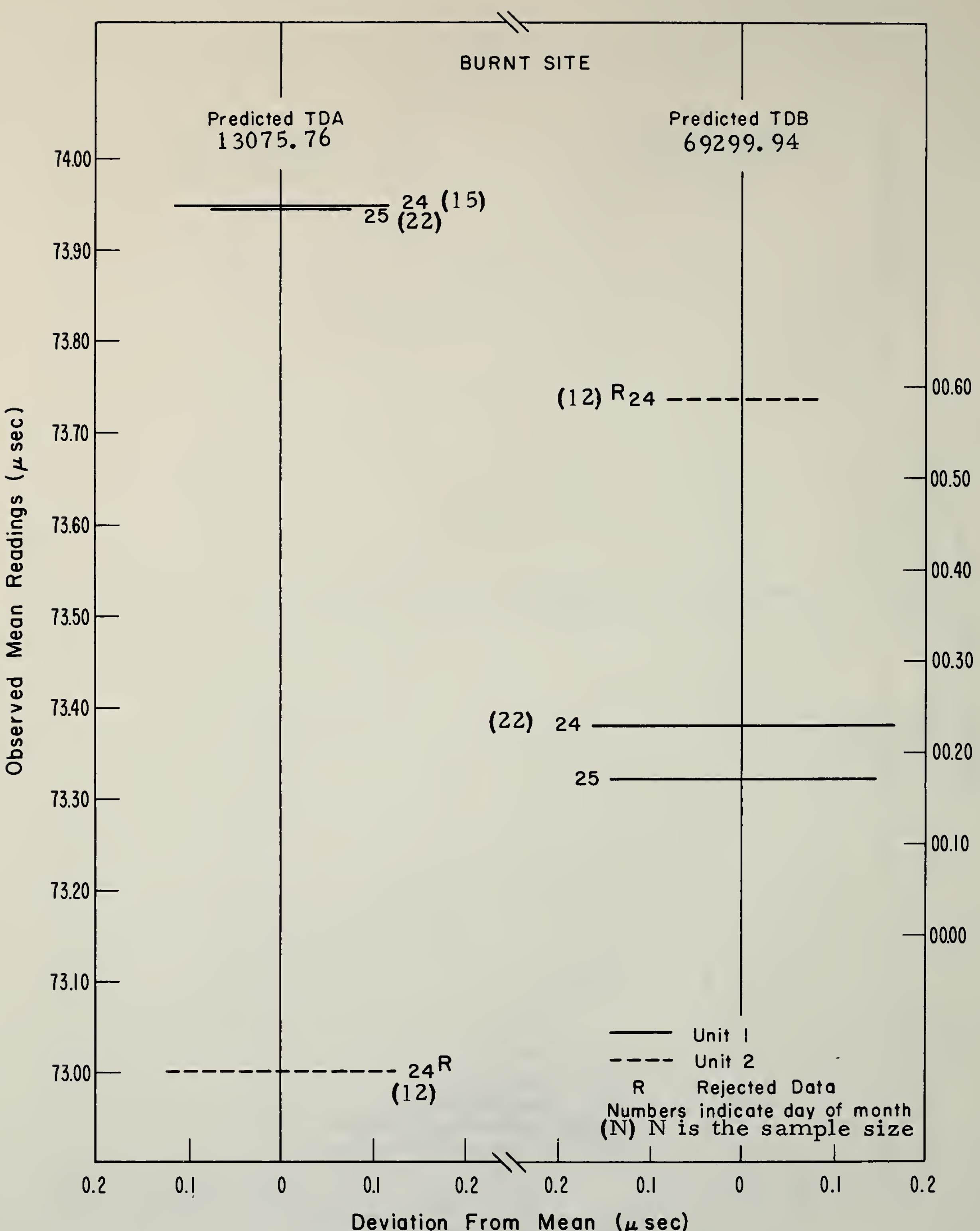

Figure 10. Data dispersion at Burnt site 1968.

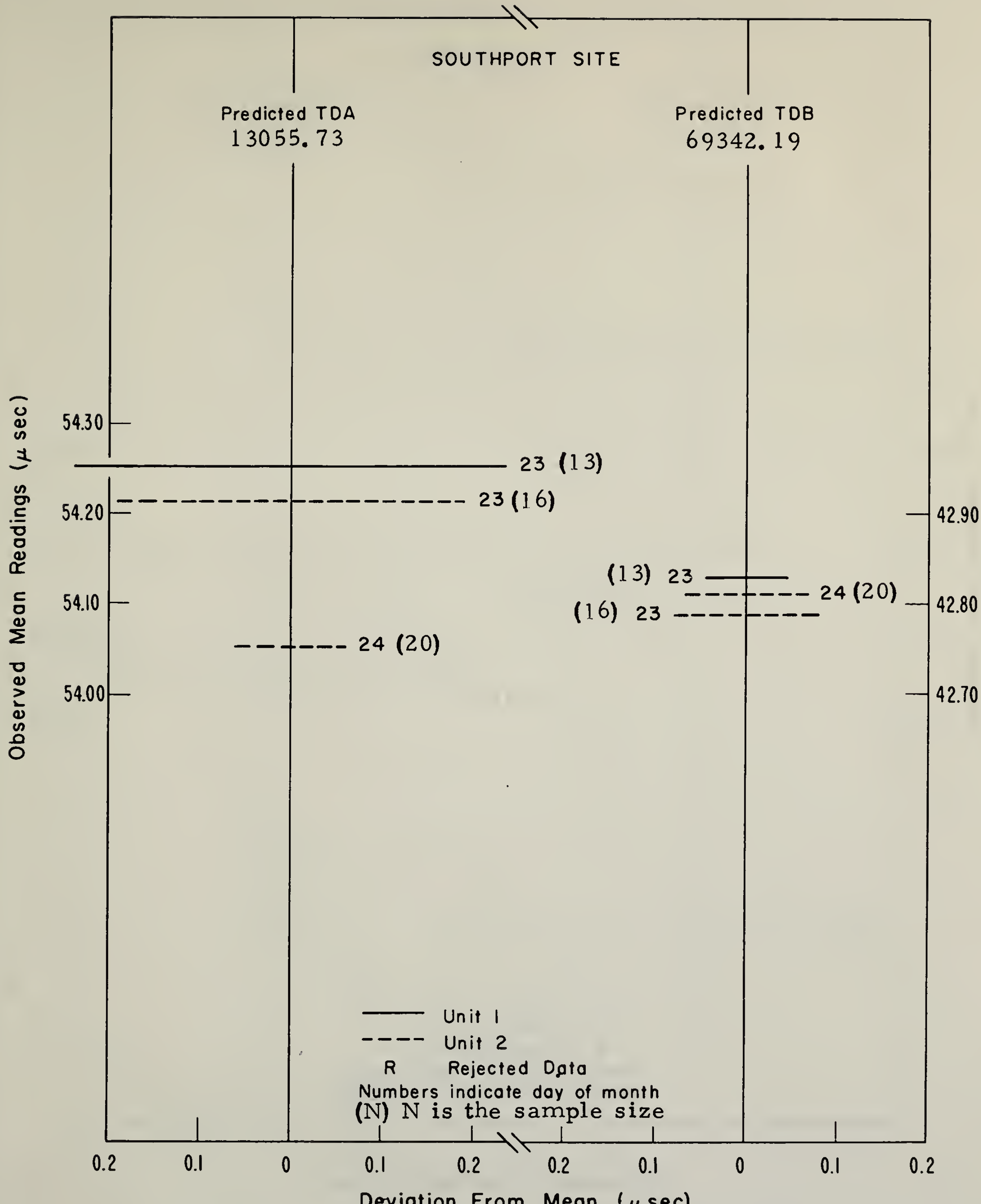

Figure 11. Data dispersion at Southport site 1968.

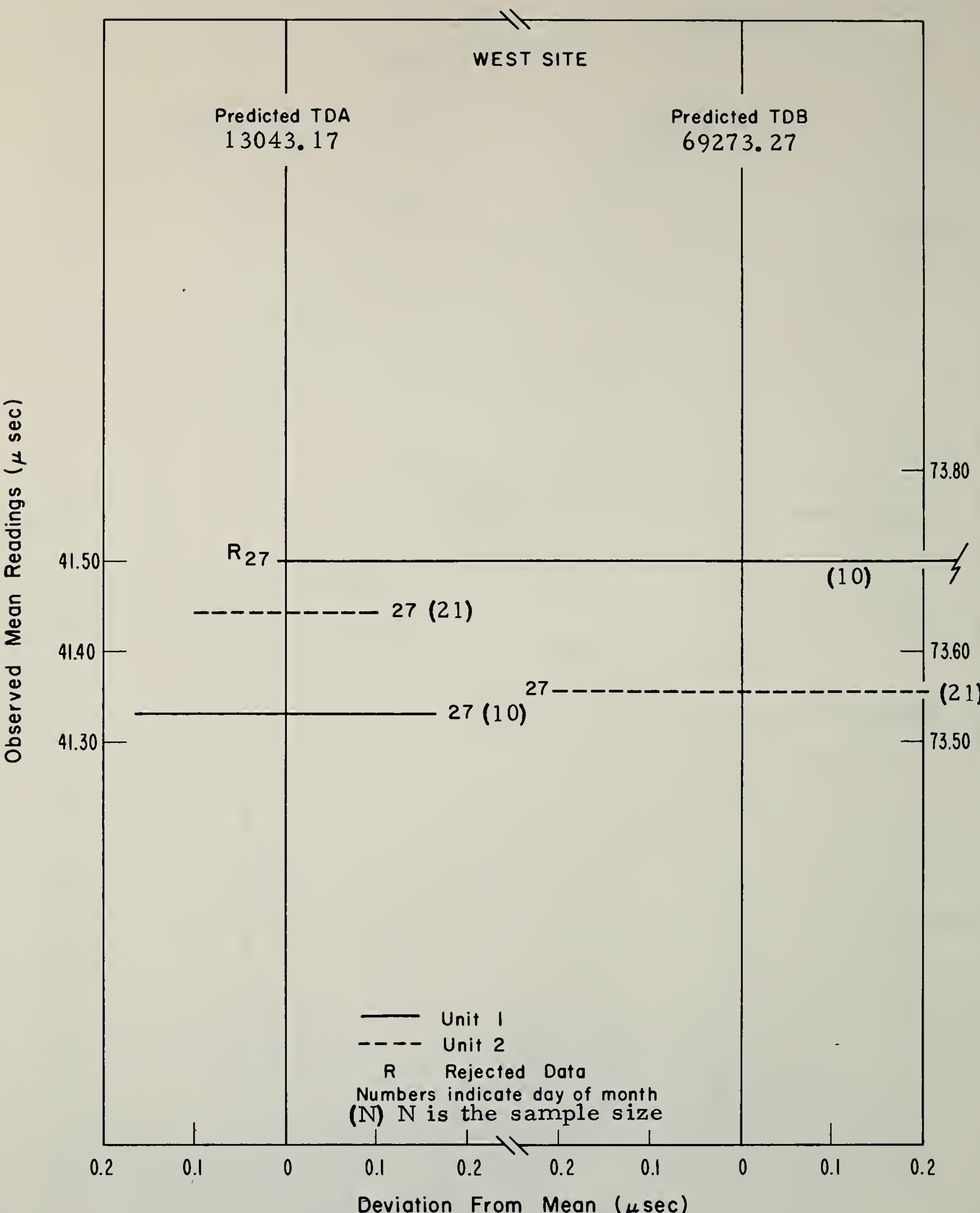

Figure 12. Data dispersion at West site 1968.

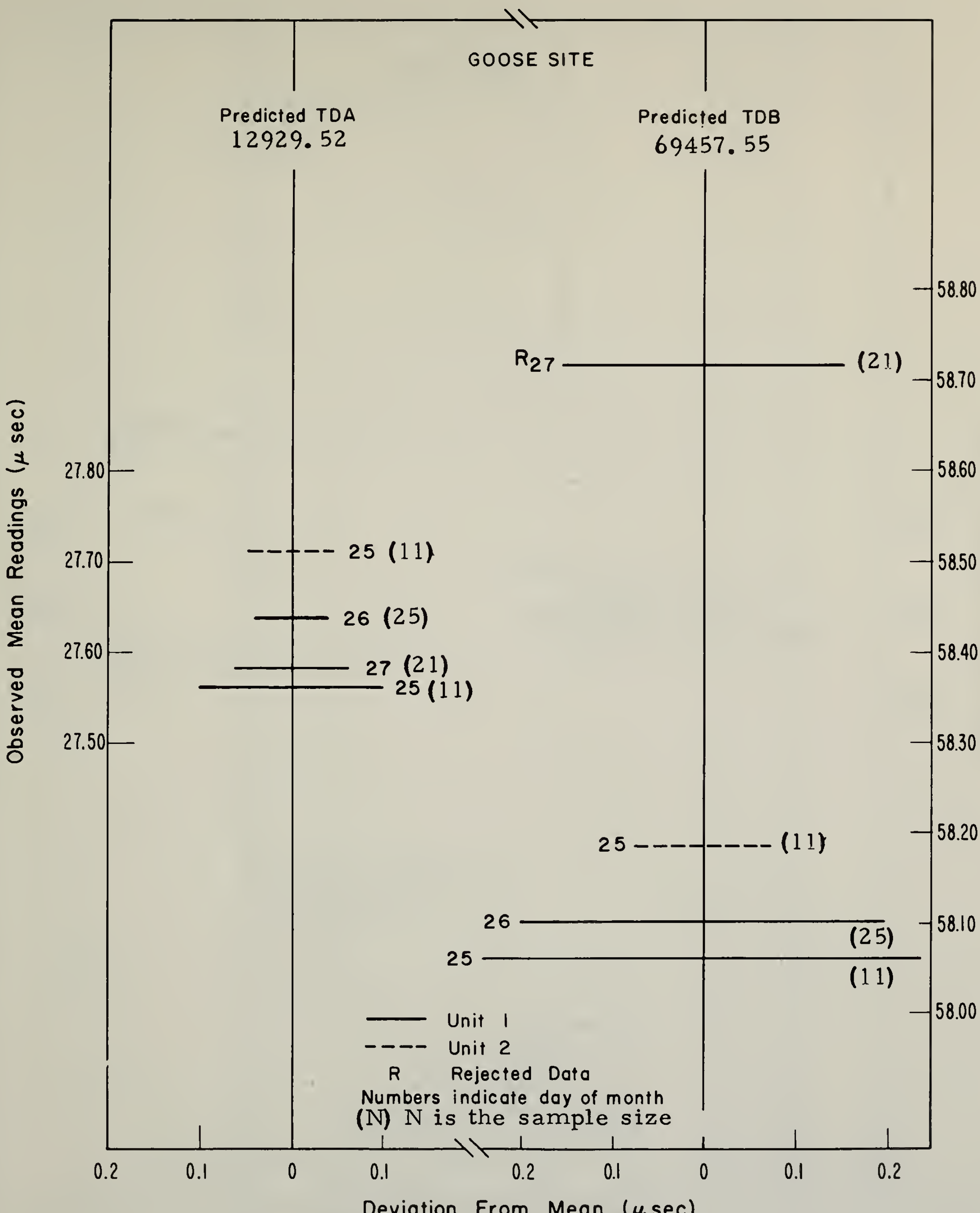

Figure 13. Data dispersion at Goose site 1968.

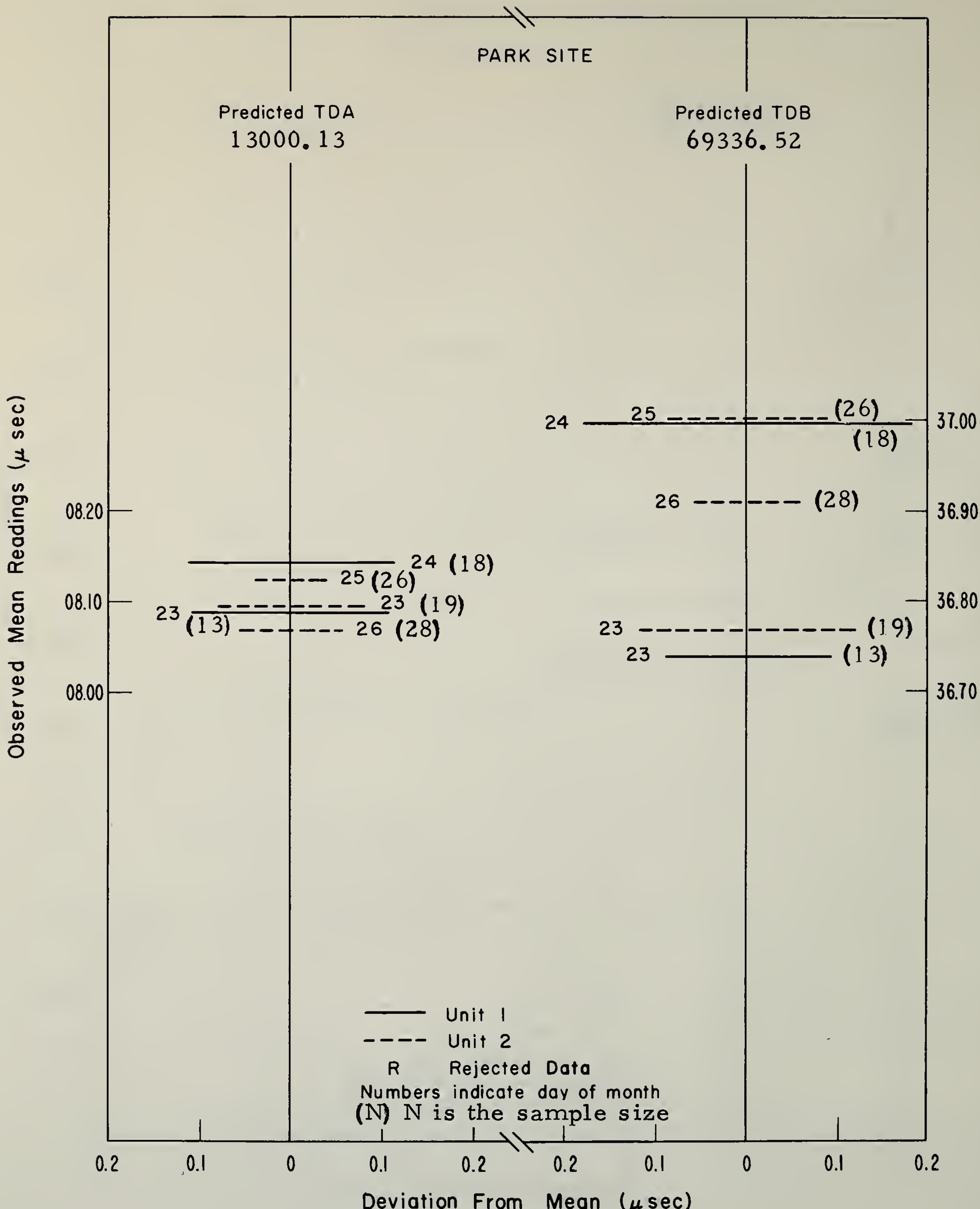

Figure 14. Data dispersion at Park site 1968.

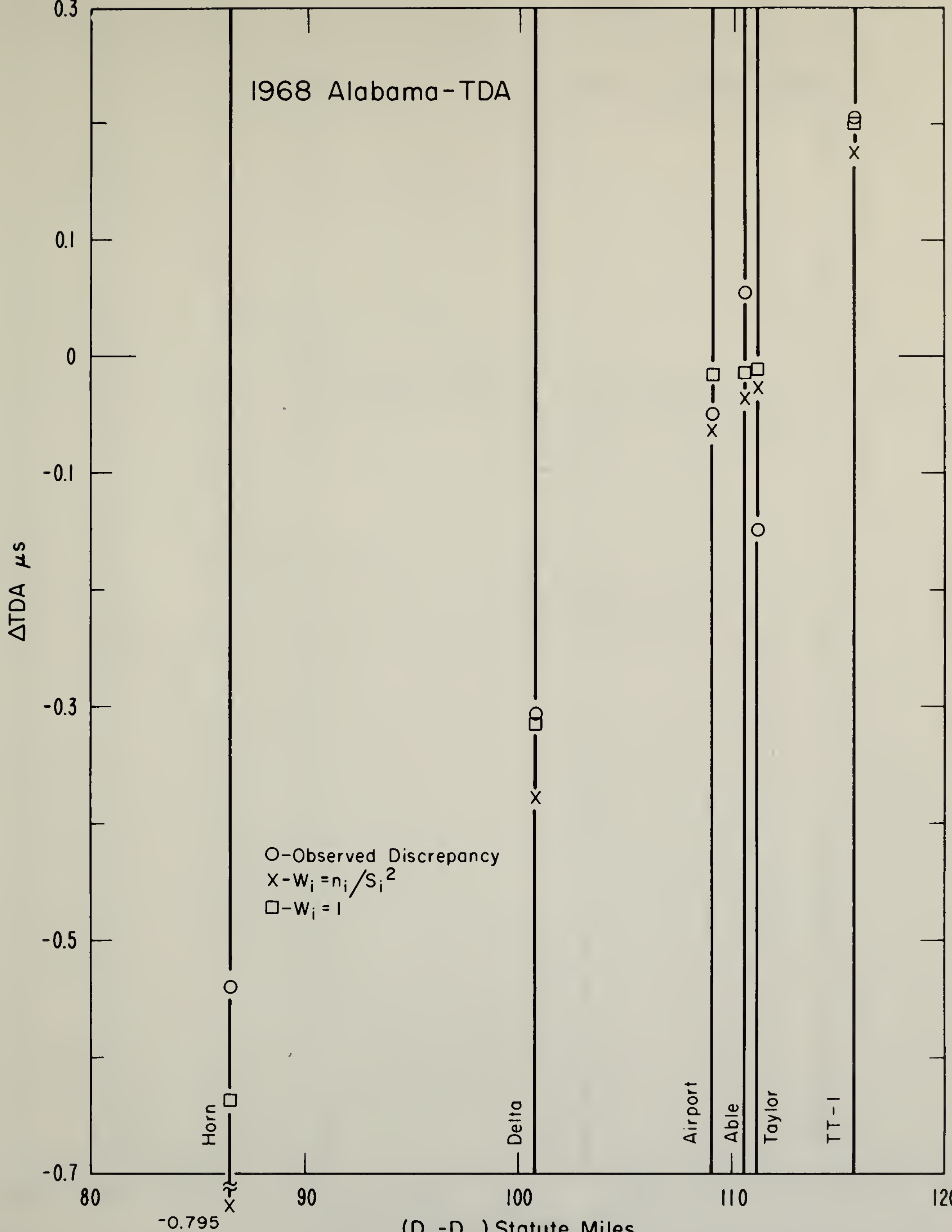

Figure 15. Least squares estimates of observed discrepancies.

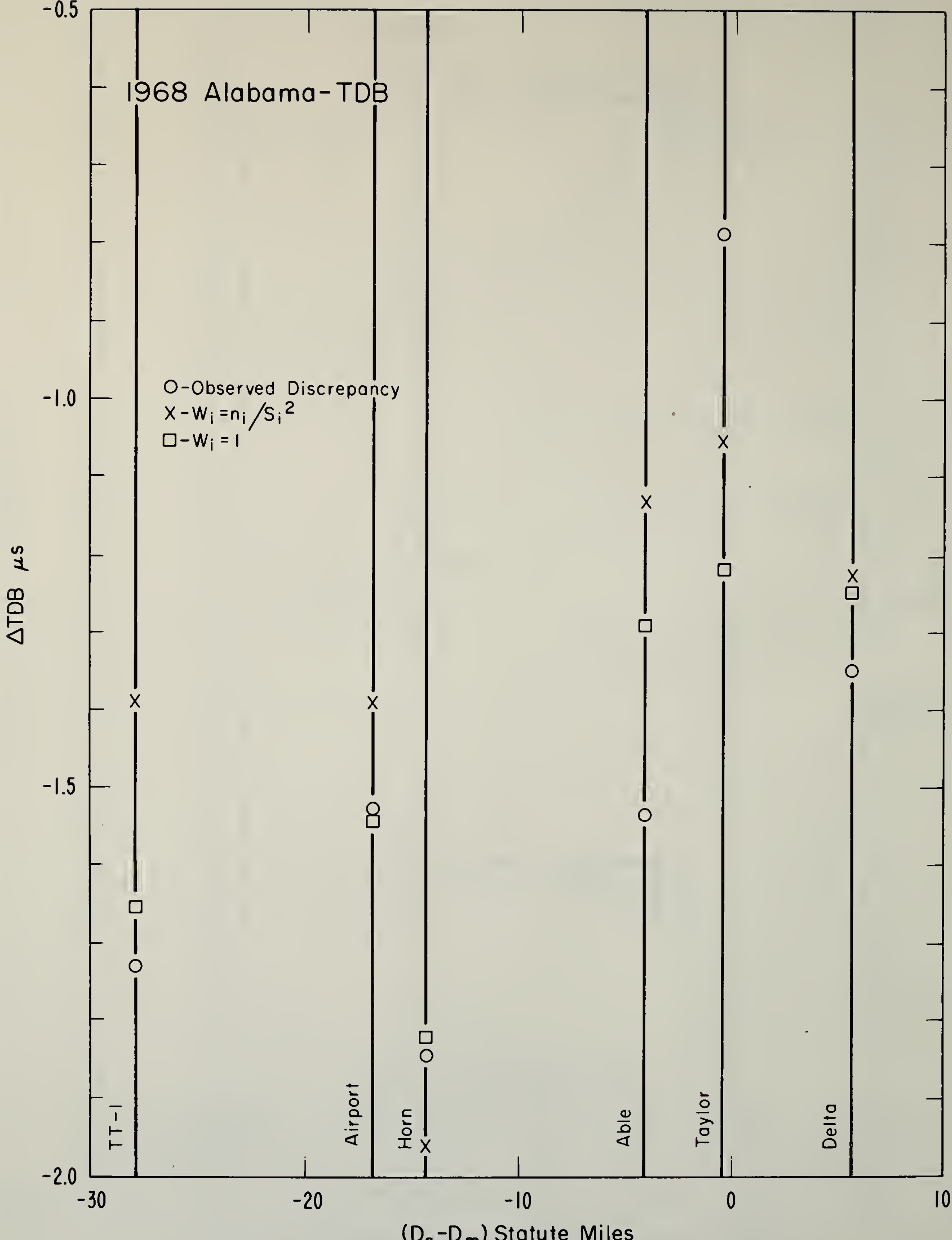

Figure 16. Least squares estimates of observed discrepancies.

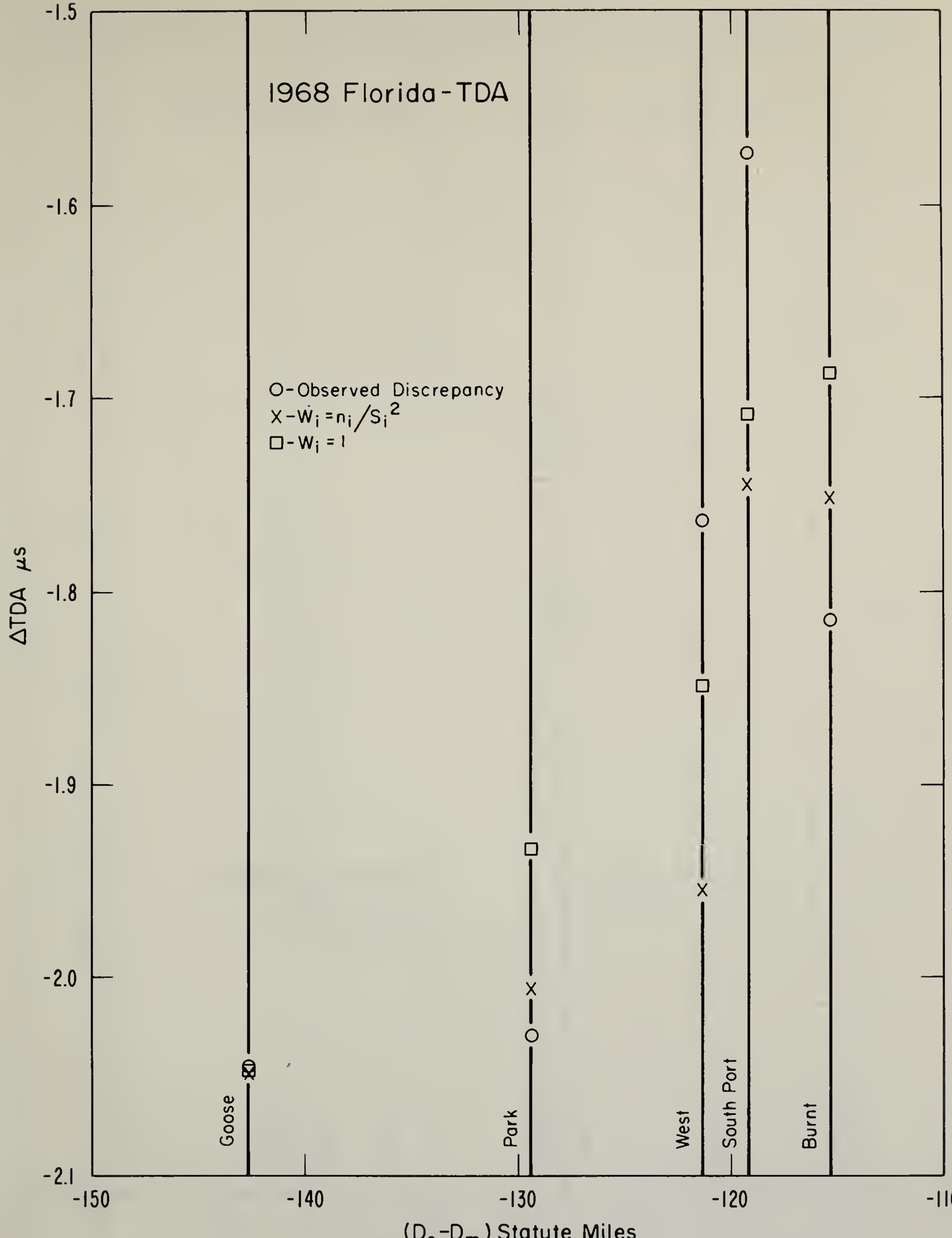

Figure 17. Least squares estimates of observed discrepancies.

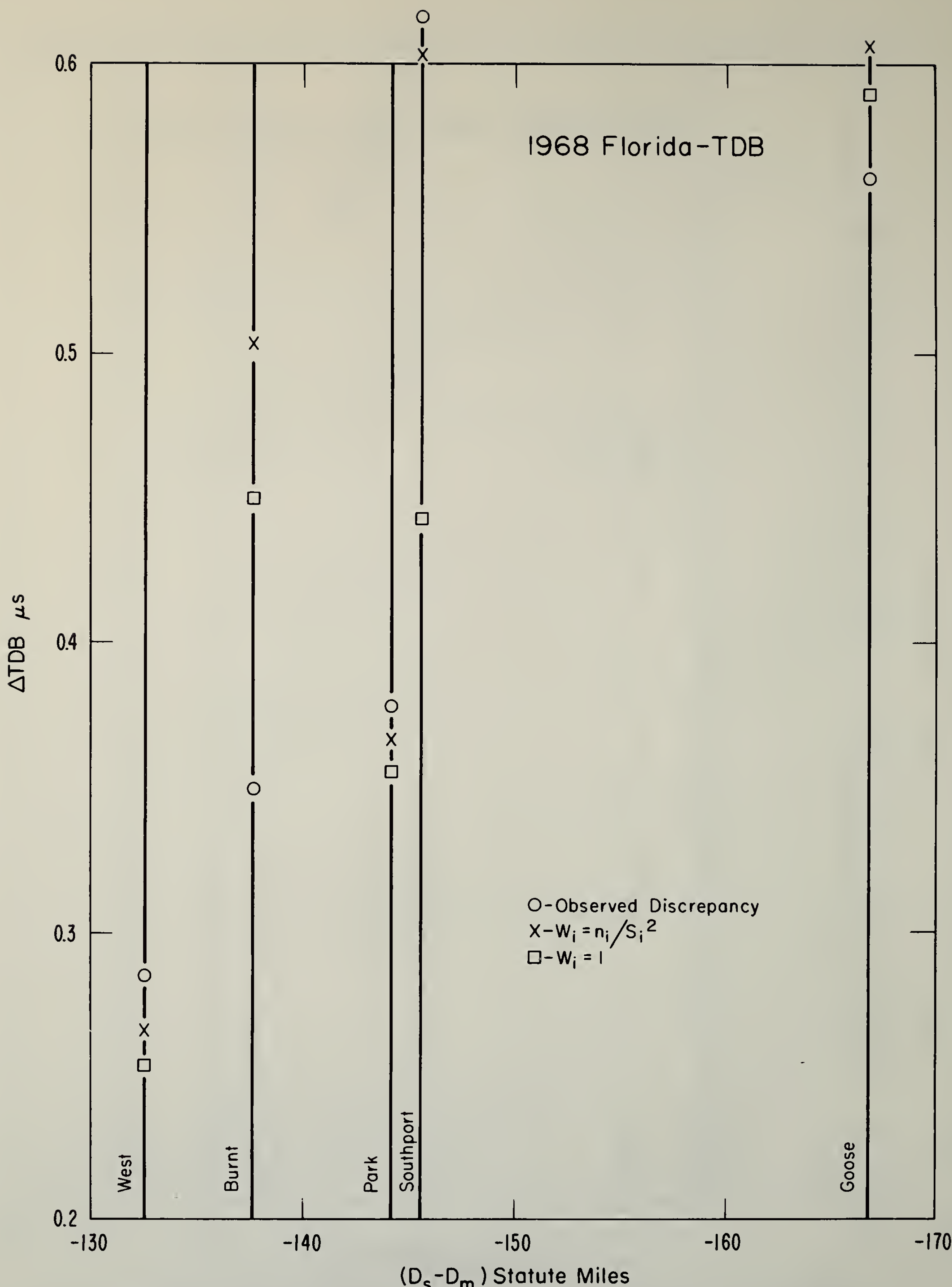

Figure 18. Least squares estimates of observed discrepancies.

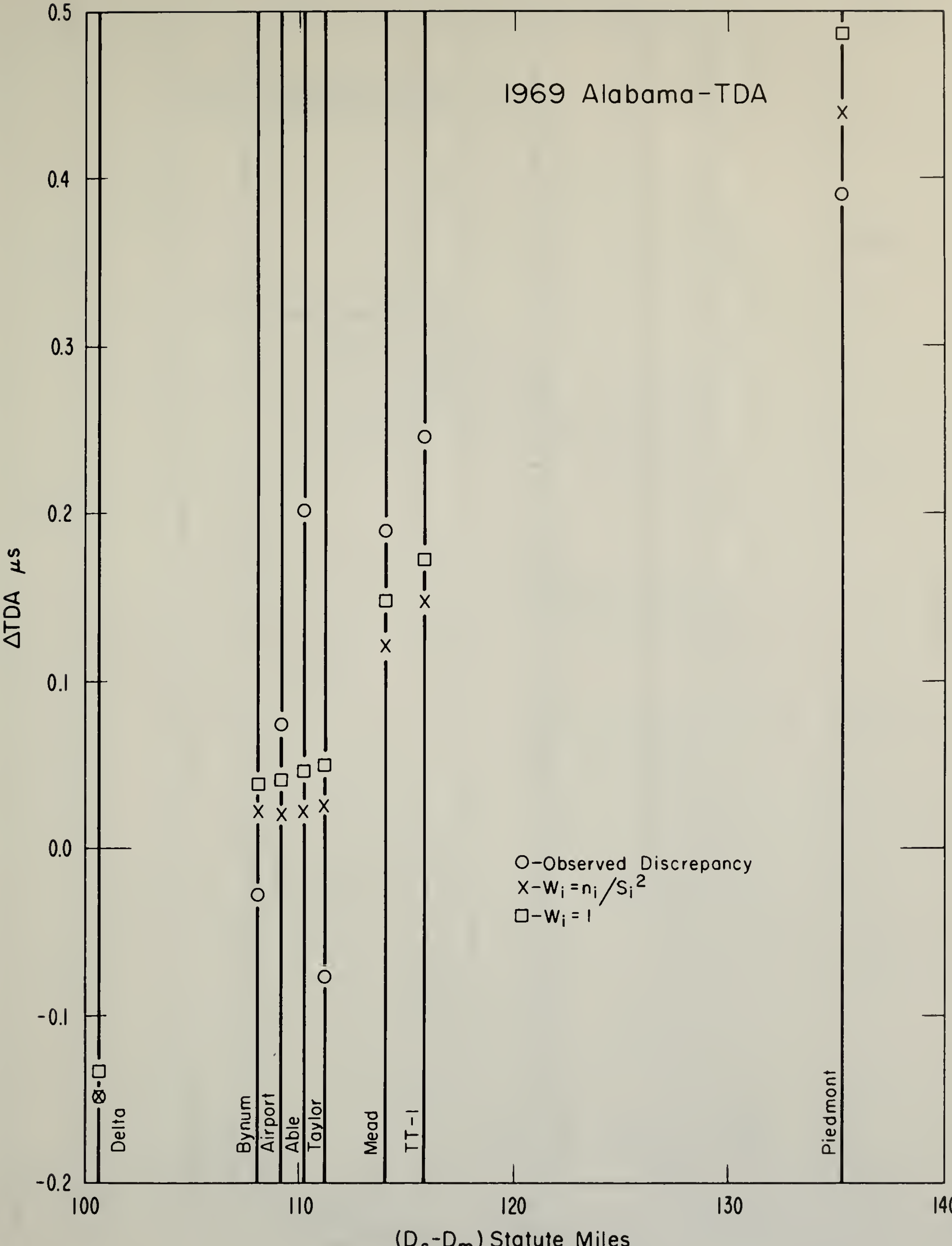

Figure 19. Least squares estimates of observed discrepancies.

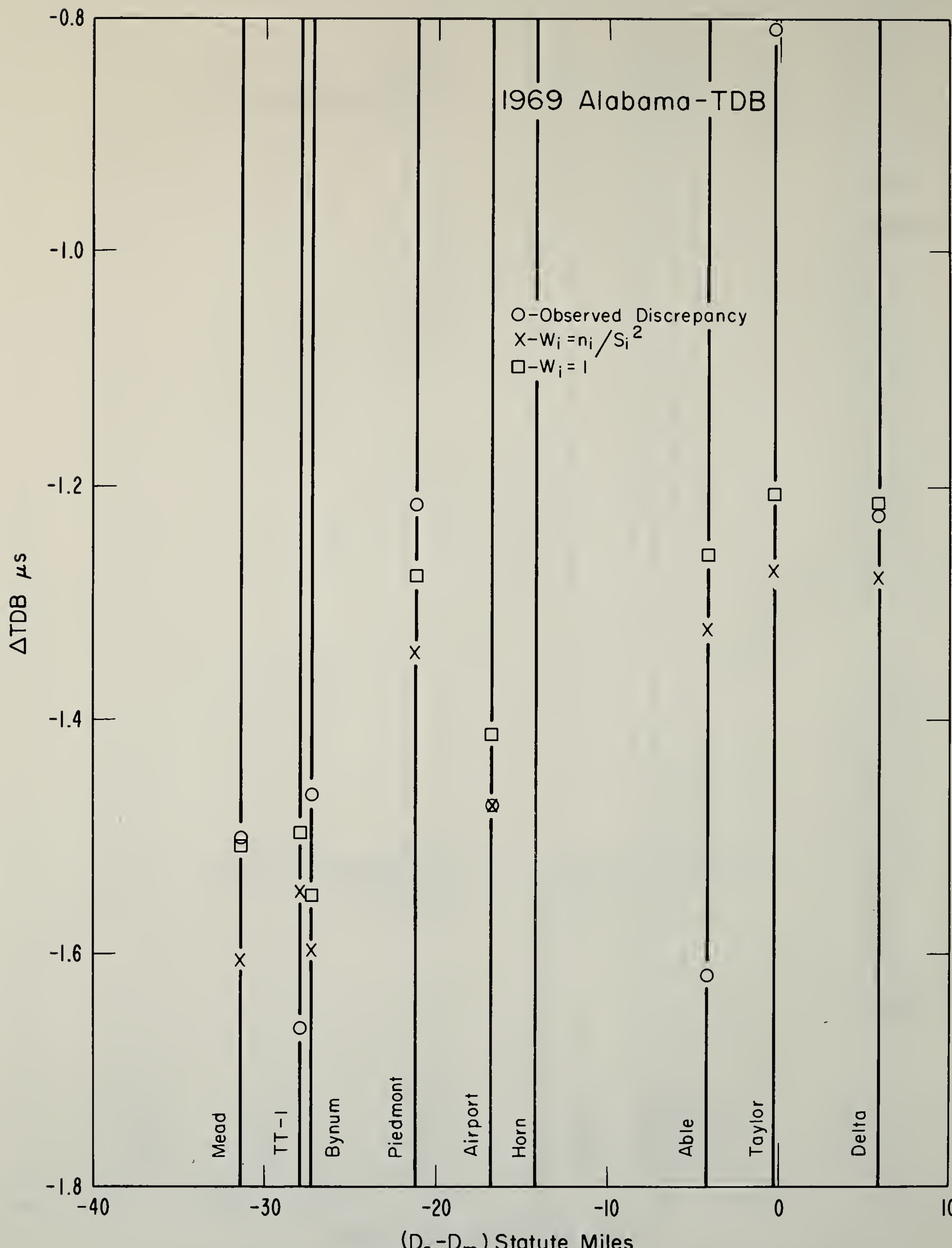

Figure 20. Least squares estimates of observed discrepancies.

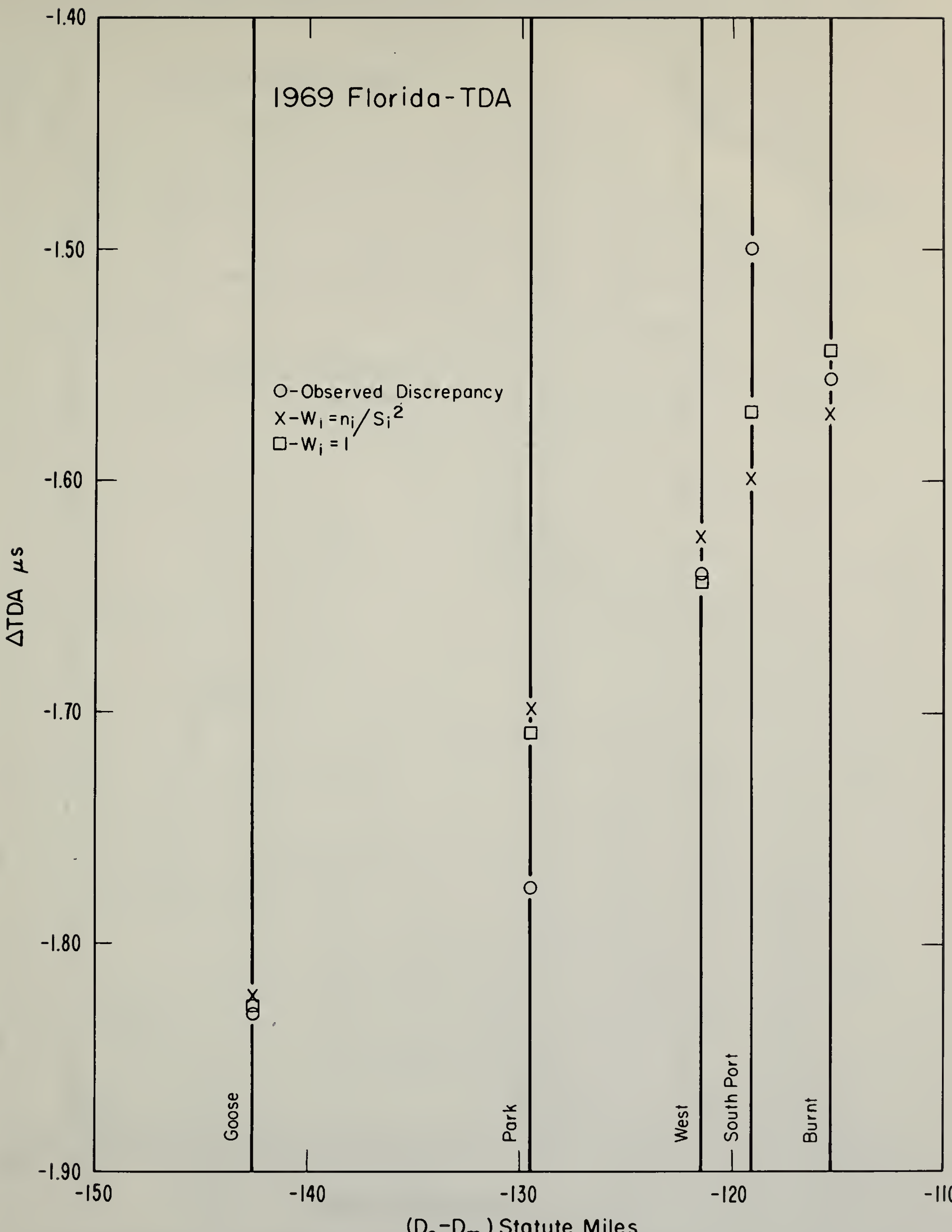

Figure 21. Least squares estimates of observed discrepancies.

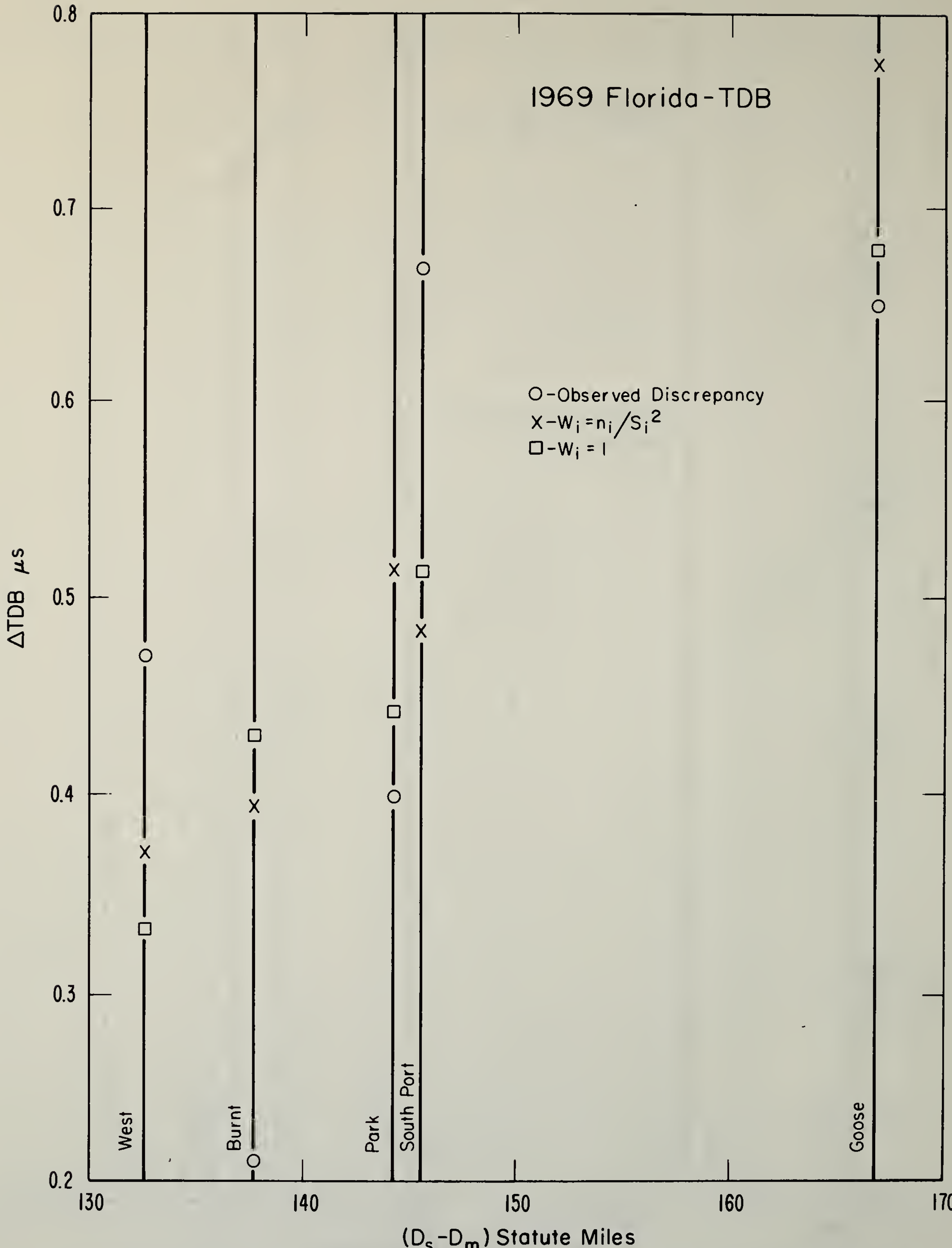

Figure 22. Least squares estimates of observed discrepancies.

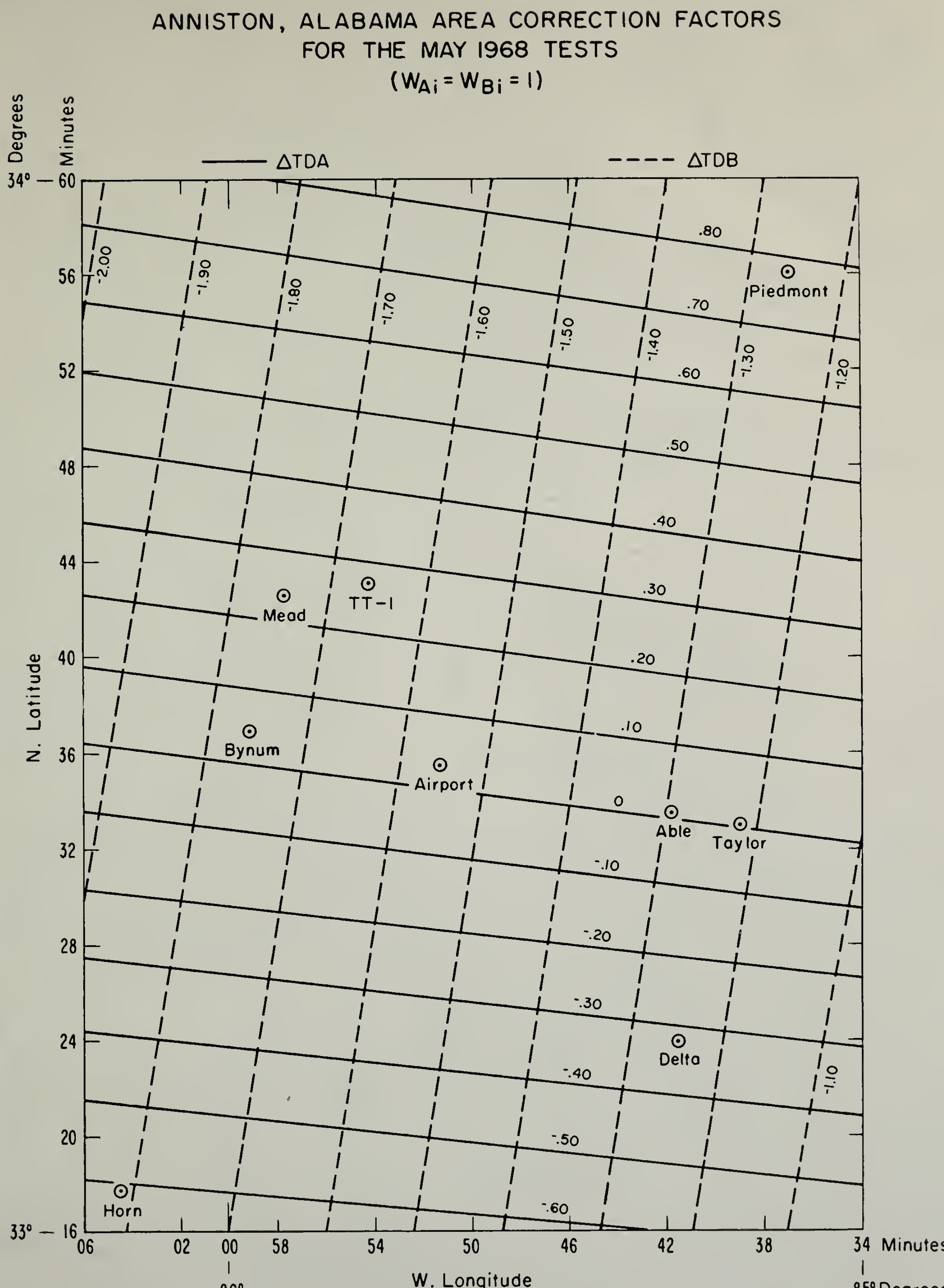

Figure 23. Least squares estimates of observed discrepancies.

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